Model Based Iterative Reconstruction With Spatially Adaptive Sinogram Weights for Wide-Cone Cardiac CT

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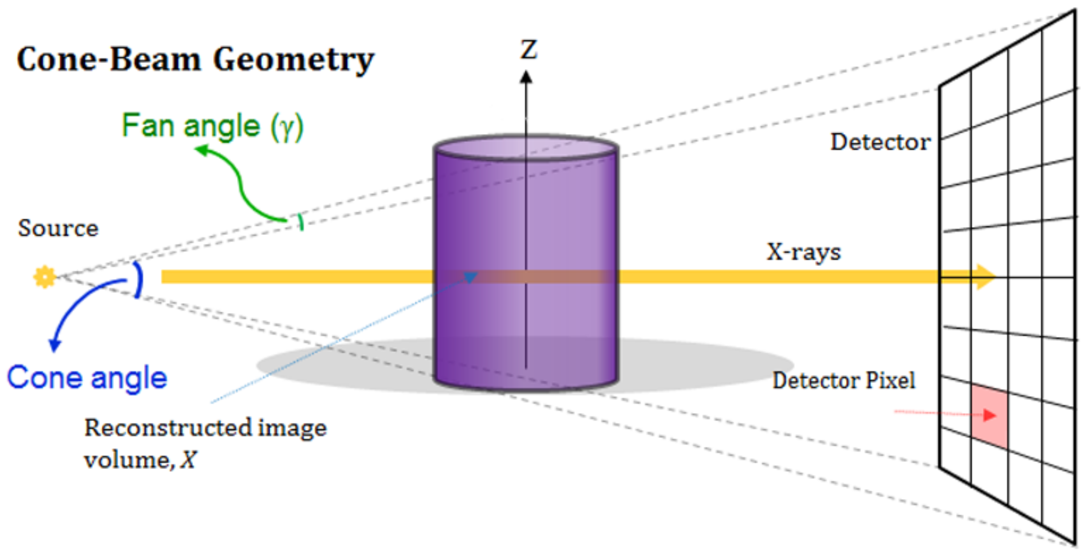
Outline

- High Temporal Resolution For Cardiac CT
- Full-scan vs Half-scan Cardiac CT
- Spatially Adaptive sinogram Weighting MBIR (SAW-MBIR)
- Results
- Conclusions

High Temporal Resolution for Cardiac CT



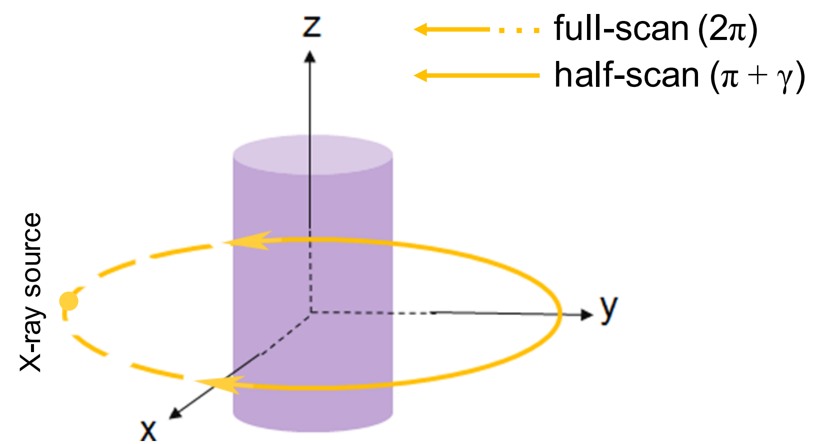
❑ Wide cone detector: image heart in a single rotation

❑ Full scan: Rotation of 2π

- Full ROI with suboptimal temporal resolution

❑ Half-scan: Rotation of $\pi + \gamma$

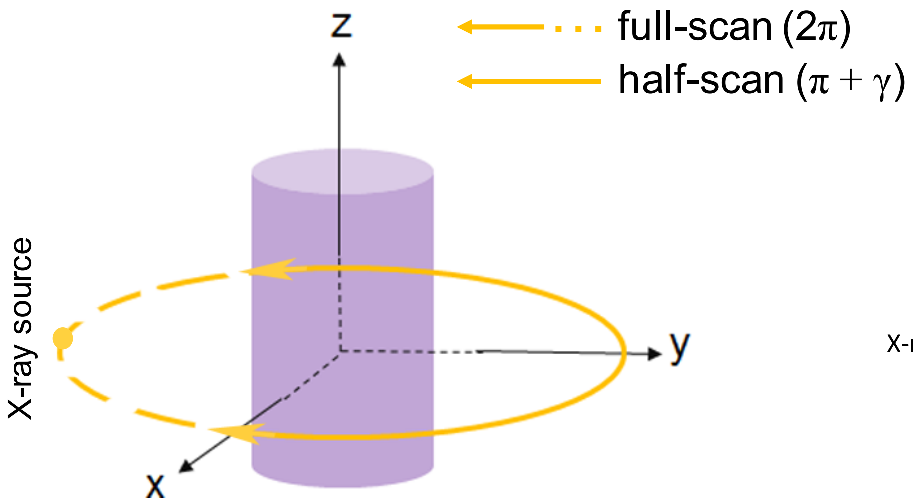
- Smaller ROI with better temporal resolution

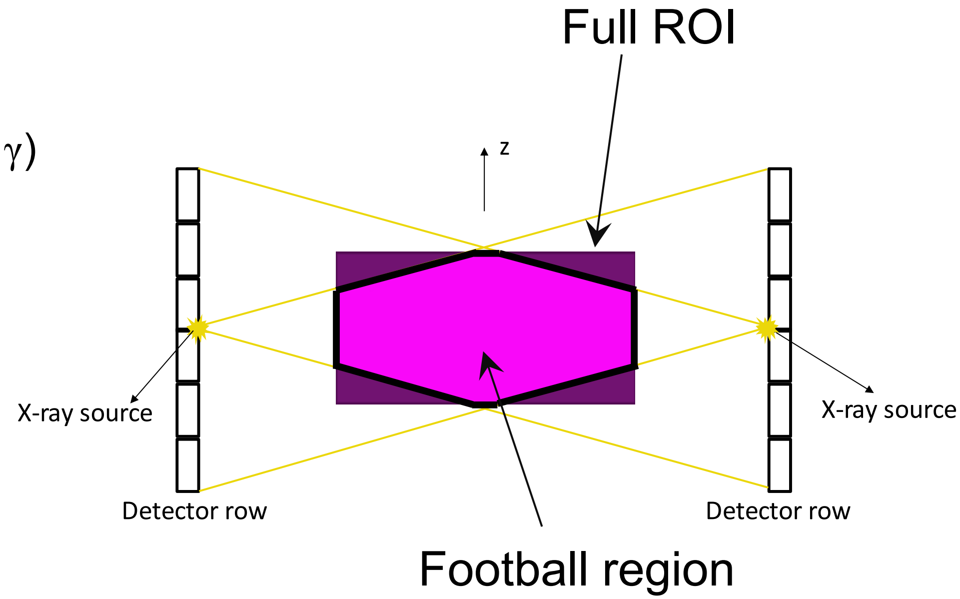


Full-Scan vs. Half-Scan Cardiac CT

- Football region:
  - Fully sampled by a half-scan

- Full ROI:
  - **<u>Not</u>** fully sampled by half-scan





Our Goals

- Reconstruct full ROI with high temporal resolution:
  - Eliminate any motion artifacts
  - Properly reconstruct edges of ROI
  - Achieve this with a single reconstruction

Existing Approaches

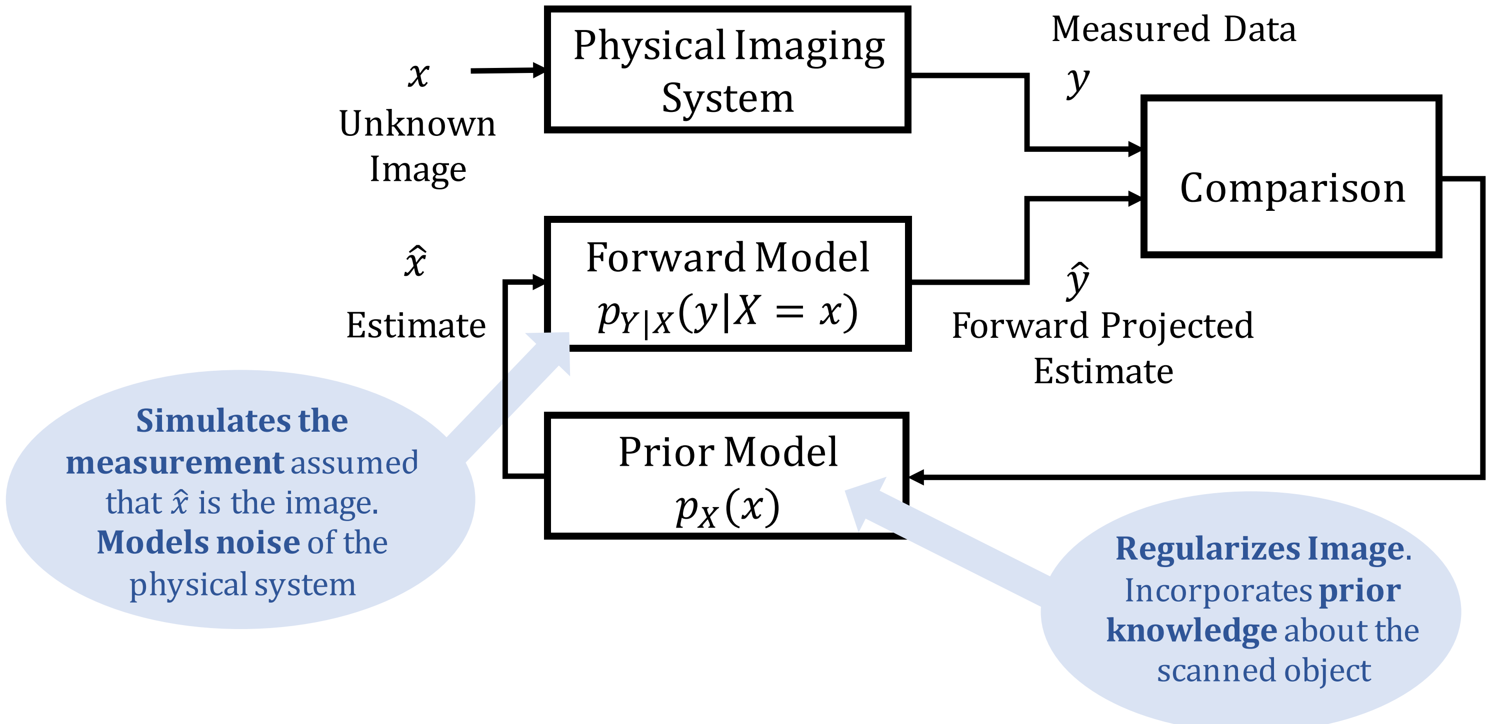
- **Several FBP based approaches exist in literature**
  - J. D. Pack, et al., Fully 3D , 2013.
  - J. Tang, et al., Medical Physics, 2010.
  - B. Chiang, et al., Nuclear Science
- **Use full-scan as prior model for half-scan recon**
  - J. H. Cho, dissertation, 2014.
- **Use additional full-scan measurements to extrapolate the half-scan measurements and statistical weights for reconstruction outside football region**
  - J. H. Cho, dissertation, 2014.
- **Separate full-scan and half-scan reconstruction and merge the results**
  - Proposed in: J. H. Cho, dissertation, 2014.
  - Computationally expensive

Our approach

- SAW-MBIR:
  Spatially Adaptive sinogram Weighting MBIR

- Idea:
  Compute MBIR reconstruction with **unmatched** forward and back projectors, where the **proposed back-projection operator selectively back-projects full-scan data to improve temporal resolution in football region and maintain high quality outside football region**

Model Based Iterative Reconstruction (MBIR)



Solution to inverse problem: **m**aximum **a** **p**osteriori estimate

$$\hat{x}_{\text{MAP}} = \underset{x \geq 0}{\arg\max} \{p_{X|Y}(x|Y)\} = \underset{x \geq 0}{\arg\min} \{-\log p_{Y|X}(Y|x) - \log p_X(x)\}$$

$$= \underset{x \geq 0}{\arg\min} \{\|Y - Ax\|_W^2 + \Phi(x)\}$$

Traditional Gradient Descent for MBIR

- $y = Ax$ $\quad\quad A \in \Re^{M \times N}, x \in \Re^{N},\ y \in \Re^{M}$
- MAP cost function:

$$\hat{x} \leftarrow \arg\min_{x} f(x) = \frac{1}{2}\|y - Ax\|_{W}^{2} + \Phi(x)$$

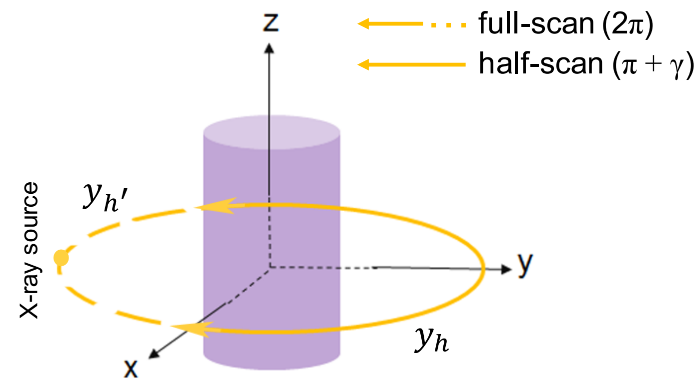
Gradient Descent Algorithm For MBIR
$y \leftarrow measured\ sinogram$ $\mathrm{x}^{0} \leftarrow FBP$ $\alpha \leftarrow step\ size$ $For\ k\ iterations\ \{$ $\quad g^{k} = \nabla f(x) = -A^{T}W\left(y - Ax^{k}\right) + \nabla\Phi\left(x^{k}\right)$ $\quad$ $Gradient\ of\ the\ Cost\ function$ $\quad x^{k+1} = x^{k} - \alpha g^{k}$ $\}$

Decomposing Data and Image

$$y = \begin{bmatrix} y_h \\ y_{h'} \end{bmatrix} - \text{full data}$$

y_h – half scan data

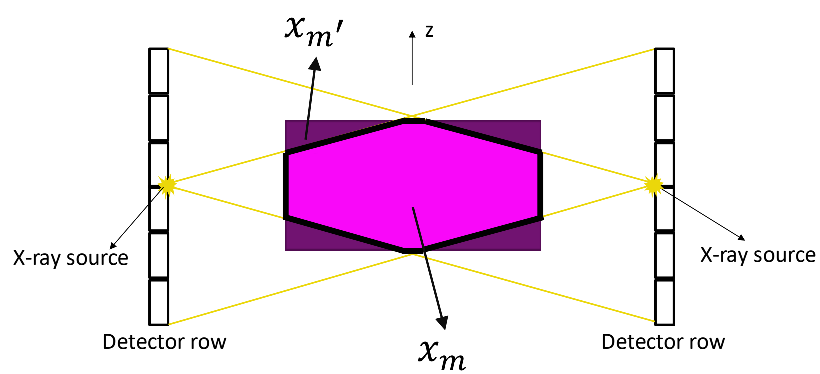
$y_{h'}$ – residual data



$$x = \begin{bmatrix} x_m \\ x_{m'} \end{bmatrix} - \text{complete image}$$

x_m – football image

$x_{m'}$ – residual image



Block Structured Forward Projector

- Forward projector

$$y = Ax$$

- Block structure forward projector

$$A = \begin{bmatrix} A_{hm} & A_{hm'} \\ A_{h'm} & A_{h'm'} \end{bmatrix}$$

where

$$\begin{array}{r} \text{Half-scan data} \rightarrow \\ \text{Residual data} \rightarrow \end{array} \begin{bmatrix} y_h \\ y_{h'} \end{bmatrix} = \begin{bmatrix} A_{hm} & A_{hm'} \\ A_{h'm} & A_{h'm'} \end{bmatrix} \begin{bmatrix} x_m \\ x_{m'} \end{bmatrix} \begin{array}{l} \leftarrow \text{Football image} \\ \leftarrow \text{Residual image} \end{array}$$

Complete Forward Projector

$$
\begin{array}{r} \\ \\ \text{Half-scan data} \longrightarrow \\ \text{Residual data} \longrightarrow \end{array}
\begin{array}{c} \begin{array}{cc} \text{Football image} & \text{Residual image} \\ \downarrow & \downarrow \end{array} \\ \begin{bmatrix} A_{hm} & A_{hm'} \\ A_{h'm} & A_{h'm'} \end{bmatrix} \end{array}
$$

Complete Back Projector

$$\begin{array}{rc}
 & \begin{array}{cc}\text{Half-scan data} & \text{Residual data}\end{array} \\
\begin{array}{r}\text{Football image} \\ \text{Residual image}\end{array} & \begin{bmatrix} A^T_{hm} & A^T_{h'm} \\ A^T_{hm'} & A^T_{h'm'} \end{bmatrix}
\end{array}$$

Masked Back Projector

$$\begin{array}{rc} & \begin{array}{cc} \text{Half-scan data} \downarrow & \text{Residual data} \downarrow \end{array} \\ \begin{array}{r} \text{Football image} \rightarrow \\ \text{Residual image} \rightarrow \end{array} & \begin{bmatrix} A_{hm}^{T} & 0 \\ A_{hm'}^{T} & A_{h'm'}^{T} \end{bmatrix} \end{array}$$

Residual data is not back projected to football image

Masked Back Projector and the Pseudo-Gradient

Key idea: Residual full scan data is not back projected to football image

$$A^T_{mask} = \begin{bmatrix} A^T_{hm} & 0 \\ A^T_{hm'} & A^T_{h'm'} \end{bmatrix}$$

"Pseudo-Gradient" is given by

$$g^k_s = -A^T_{mask} W(y - Ax) + \nabla\Phi(x)$$

SAW-MBIR

- **New SAW-MBIR algorithm: Spatially Adaptive sinogram Weighting MBIR**

$$A_{mask}^{T} = \begin{bmatrix} A_{hm}^{T} & 0 \\ A_{hm'}^{T} & A_{h'm'}^{T} \end{bmatrix}$$

Pseudo-Gradient Descent Algorithm
$y \leftarrow measured\ sinogram$ $\mathrm{x}^{0} \leftarrow FBP$ $\alpha \leftarrow step\ size$ $For\ k\ iterations\ \{$ $\quad g_s^k = -A_{mask}^{T} W(y - Ax) + \nabla\Phi(x) \qquad pseudo - Gradient$ $\quad x^{k+1} = x^k - \alpha g_s^k$ $\}$

Convergence

- **This algorithm does not minimize a cost function**

- **Any fixed point, x, has the properties that**
  - $A_{mask}^{T}(y - Ax) = \nabla\Phi(x)$
  - Interpretation: Prior gradient balances pseudo-gradient

- **Convergence**
  - Empirically observed to converge
  - Sufficient condition: Converges if f is a contraction mapping where

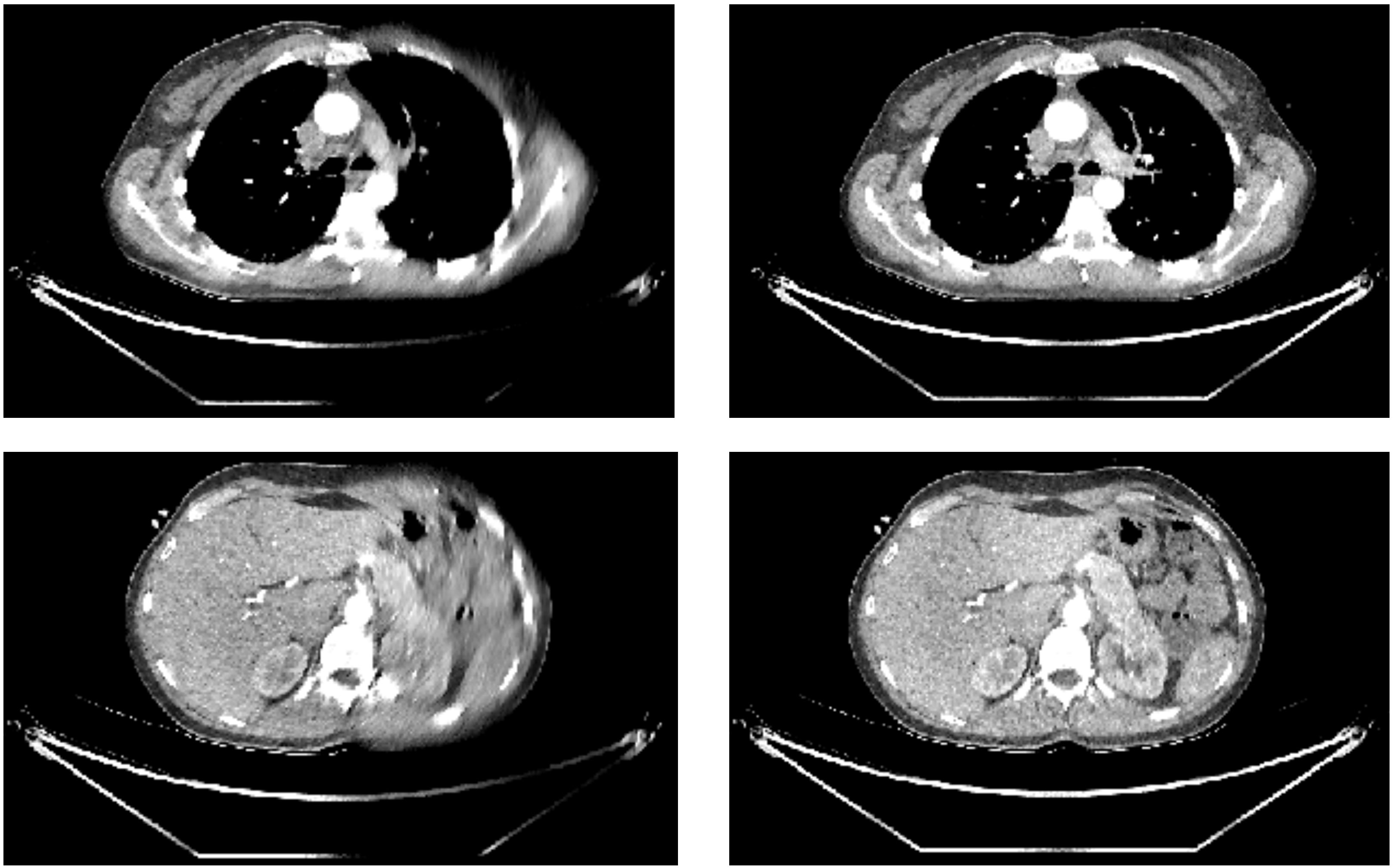
$$f(x) = x + \alpha\left[A_{mask}^{T} W(y - Ax) - \nabla\Phi(x)\right]$$

Artifact Reduction Outside Football

Half-scan | SAW-MBIR

Top slice

bottom slice



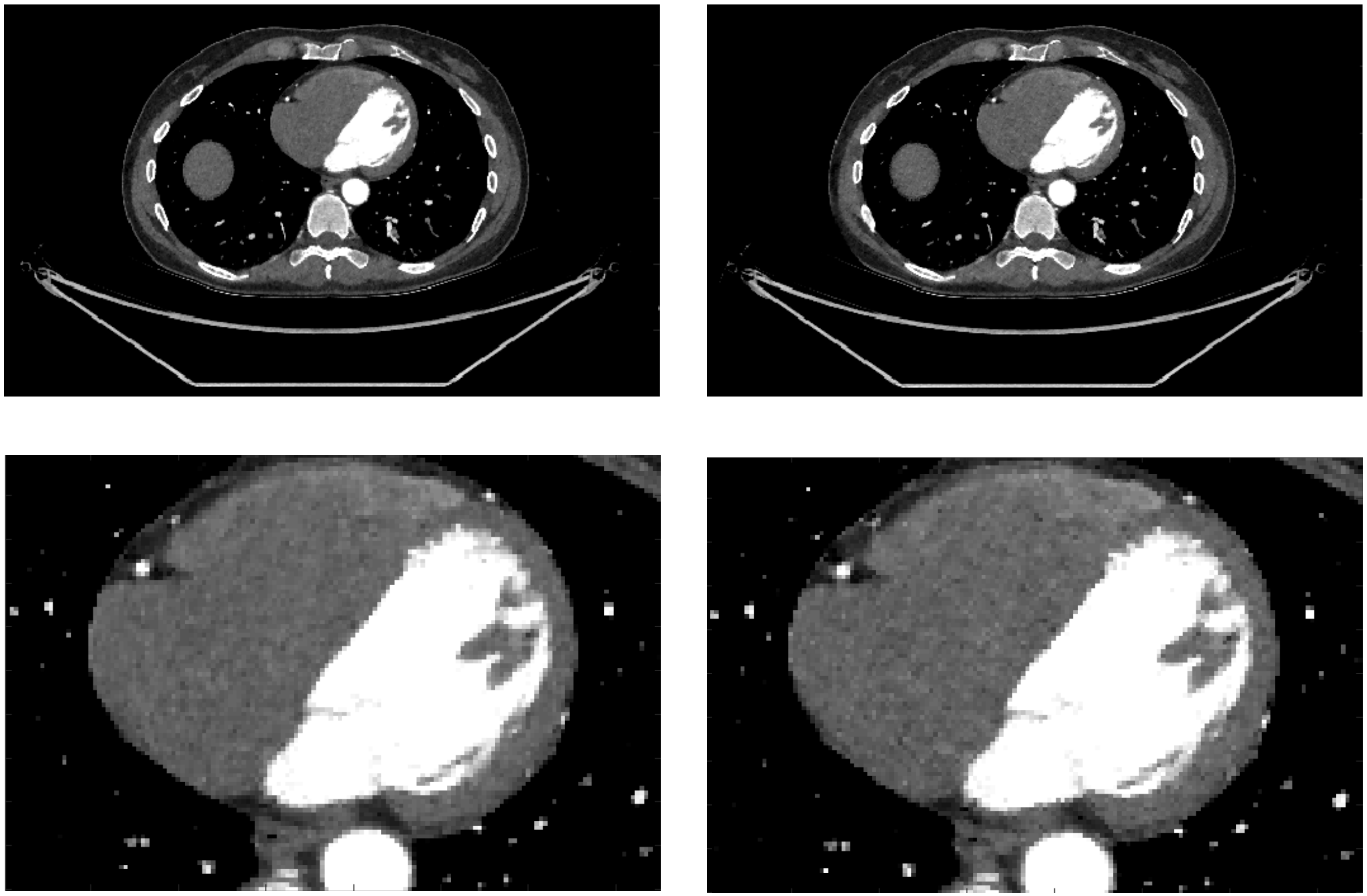
Temporal Resolution Inside Football

Half-scan

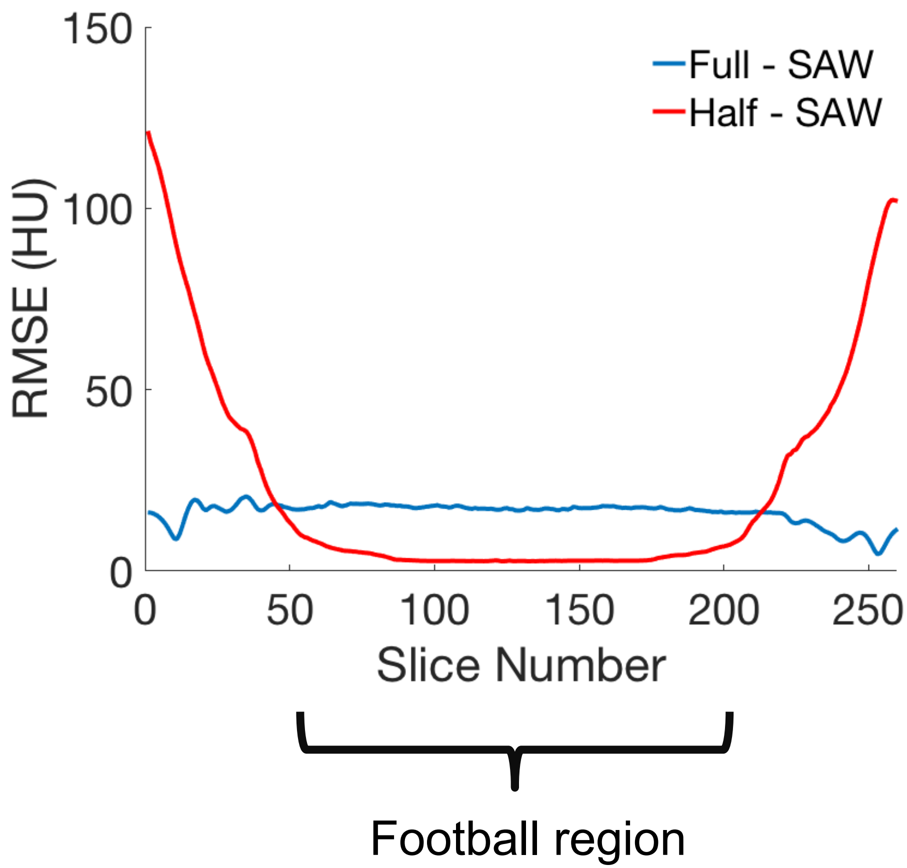
SAW-MBIR

Center slice

Center slice



Achieves best of Half and Full Scan Recons



- SAW-MBIR:
  - Matches half-scan recon inside football
  - Matches full-scan recon outside football

Conclusions

- **SAW-MBIR:**
  - Maintains same temporal resolution as half-scan MBIR
  - Maintains same image quality as full-scan MBIR at the edge slices
  - Empirically observed to converge to a fixed point

- **Future directions:**
  - Apply to spatially localized image artifacts/degradations
  - Extend theory to better understand convergence properties

Thank You