Projected Multi-Agent Consensus Equilibrium (PMACE) for Ptychographic Image Reconstruction

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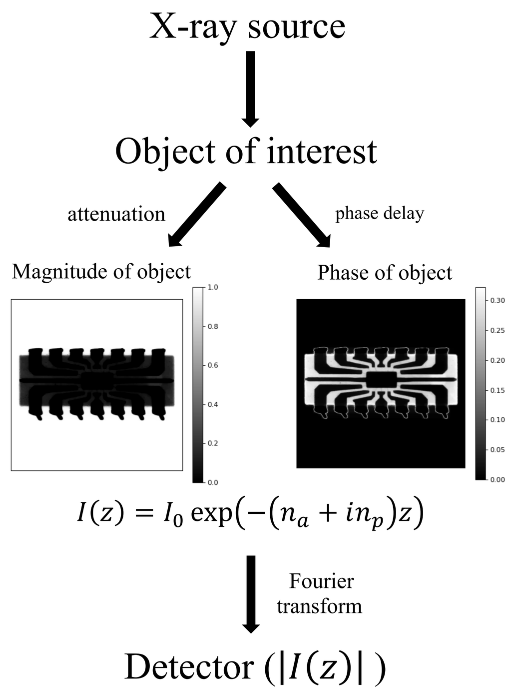
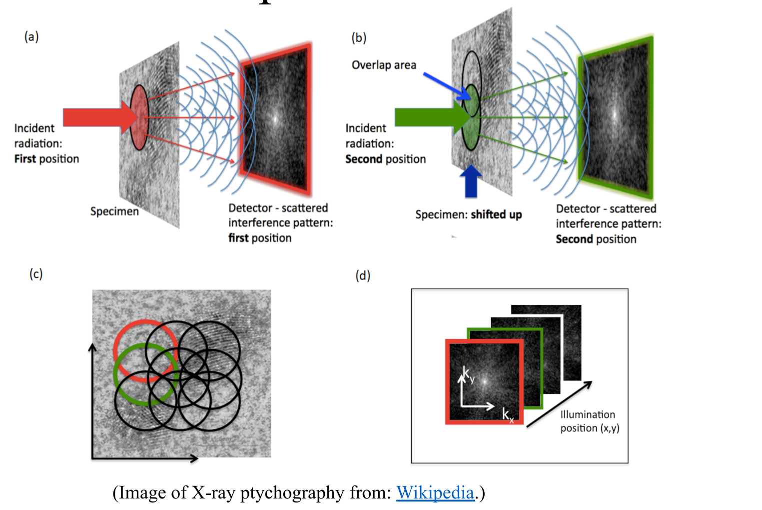
Introduction

- **Phase Contrast X-ray Imaging**
  - Detector record amplitude information in far-field plane.
  - The phase shift caused by object can not be measured directly.



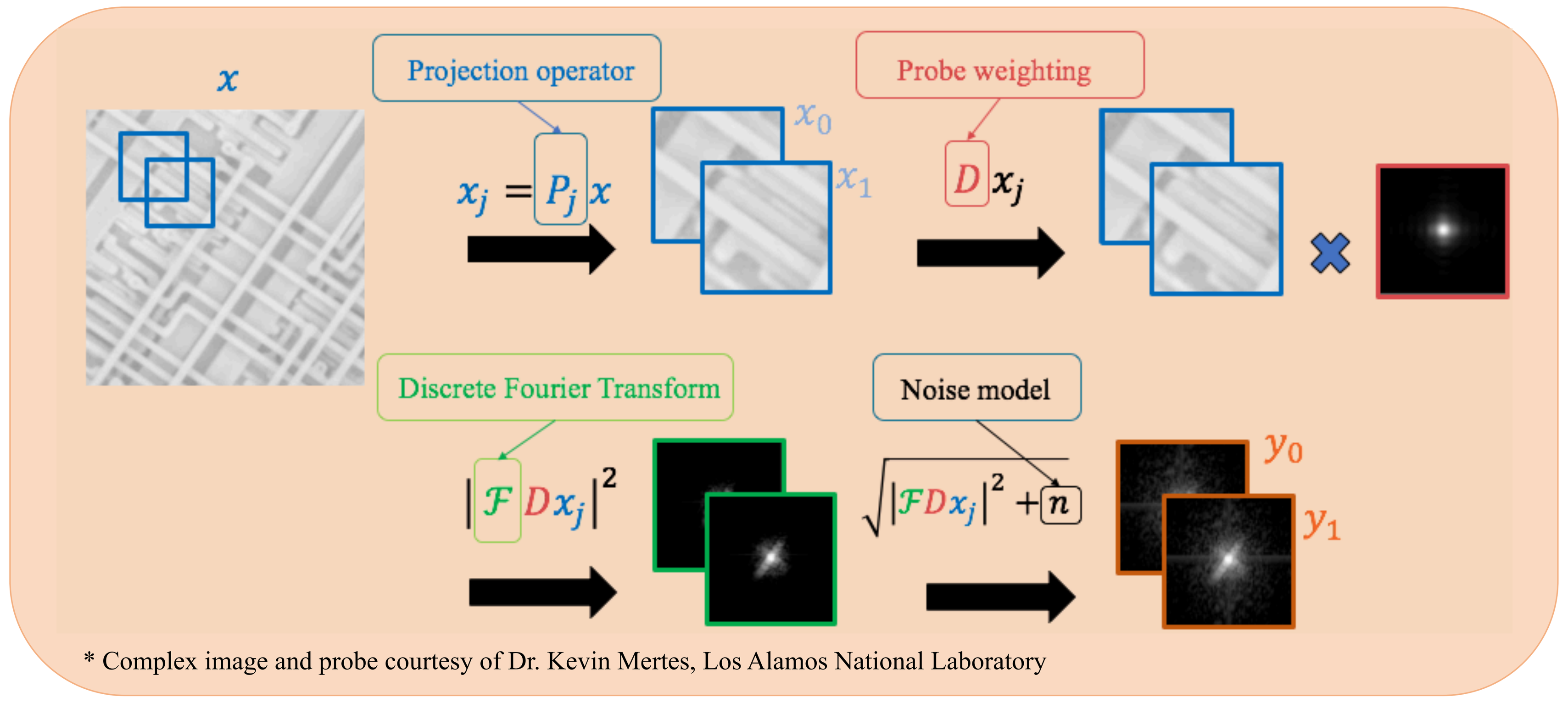
- **Ptychography Technique**
  - Makes overlaps in the field-of-view and thus introduces redundancy to recover the phase.



- **Goal**
  - Uses multiple coherent diffraction patterns to solve the nonlinear phase problem and generate the complex object.

- **Forward model:**
  - Takes the complex object and generates the resulting measurements.

$$y_j = \sqrt{|\mathcal{F} D P_j x|^2 + n} = \sqrt{|\mathcal{F} D x_j|^2 + n}$$



Key Novelties

- **Single Patch Loss Function**
  - Each loss term compares the measurement $y_j^{measure}$ with Fourier transform of corresponding projection x_j.

$$f_j(x_j) = \min_{\theta_j} \left\{ \frac{1}{2\sigma_n^2} \left\| y_j^{measure} e^{i\theta_j} - \mathcal{F} D x_j \right\|^2 \right\}, e^{i\theta_j} = \frac{\mathcal{F} D x_j}{|\mathcal{F} D x_j|}$$

θ_j matches the phase of $\mathcal{F} D x_j$

- **Update Agent**
  - The probe-weighted proximal map interpolates between current estimate and closest data-fitting point.
  - Each agent takes the one patch and produces a new estimate.

$$x_j \leftarrow F_j(x_j) = \arg\min_v \left\{ f_j(v) + \frac{1}{2\sigma^2} \| D v - D x_j \|^2 \right\}$$

$$= \frac{\alpha x_j + D^{-1} \mathcal{F}^* \left(y \frac{\mathcal{F} D x_j}{|\mathcal{F} D x_j|} \right)}{\alpha + 1}$$

Current estimate

Data-fitting point

$\alpha = \frac{\sigma_n^2}{\sigma^2}$ = Noise-to-Signal Ratio

- **Forward Operator**
  - A stack of probe-weighted proximal map.
  - It takes each of the individual patches and moves them closer to data-fitting points.

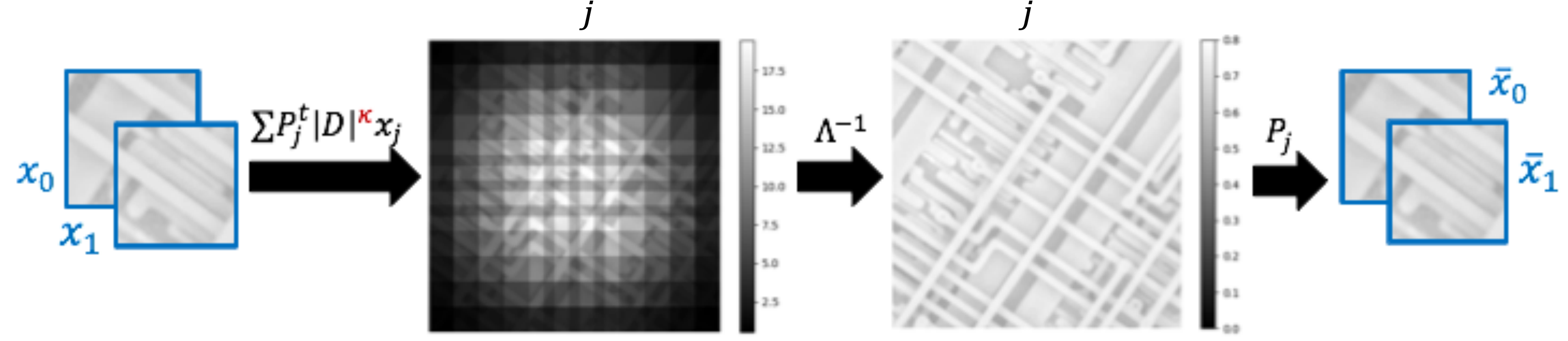
$$\boldsymbol{F}(\boldsymbol{x}) = \begin{bmatrix} F_0(x_0) \\ \vdots \\ F_{J-1}(x_{J-1}) \end{bmatrix}$$

- **Consensus Operator**
  - Combines patches with weighted average and re-extracts them.

$$\boldsymbol{G}(\boldsymbol{x}) = \begin{bmatrix} \bar{x}_0 \\ \vdots \\ \bar{x}_{J-1} \end{bmatrix}$$

  - To weight the overlapping areas appropriately, we consider
    - Number of projections
    - Probe weighting with exponent κ

$$\bar{x}_j = P_j \Lambda^{-1} \sum_j P_j^t |D|^{\kappa} x_j \text{ and } \Lambda = \sum_j P_j^t |D|^{\kappa}$$



- **Projected Multi-Agent Consensus Equilibrium (PMACE)**
  - Each agent improves estimate of corresponding patch.
  - Consensus operator makes sure the overlapping patches are consistent.
  - PMACE formulation finds the equilibrium between agents.
  - PMACE inherits the convergence property from MACE framework[1].
  - PMACE solution is solved by

$$\boldsymbol{F}(\boldsymbol{x}^*) = \boldsymbol{G}(\boldsymbol{x}^*)$$

  - Then reconstruction result is given by

$$\Lambda^{-1} \sum_j P_j^t |D|^{\kappa} \boldsymbol{x}_j^*$$

Results

- PMACE achieves better image quality than competing algorithms.

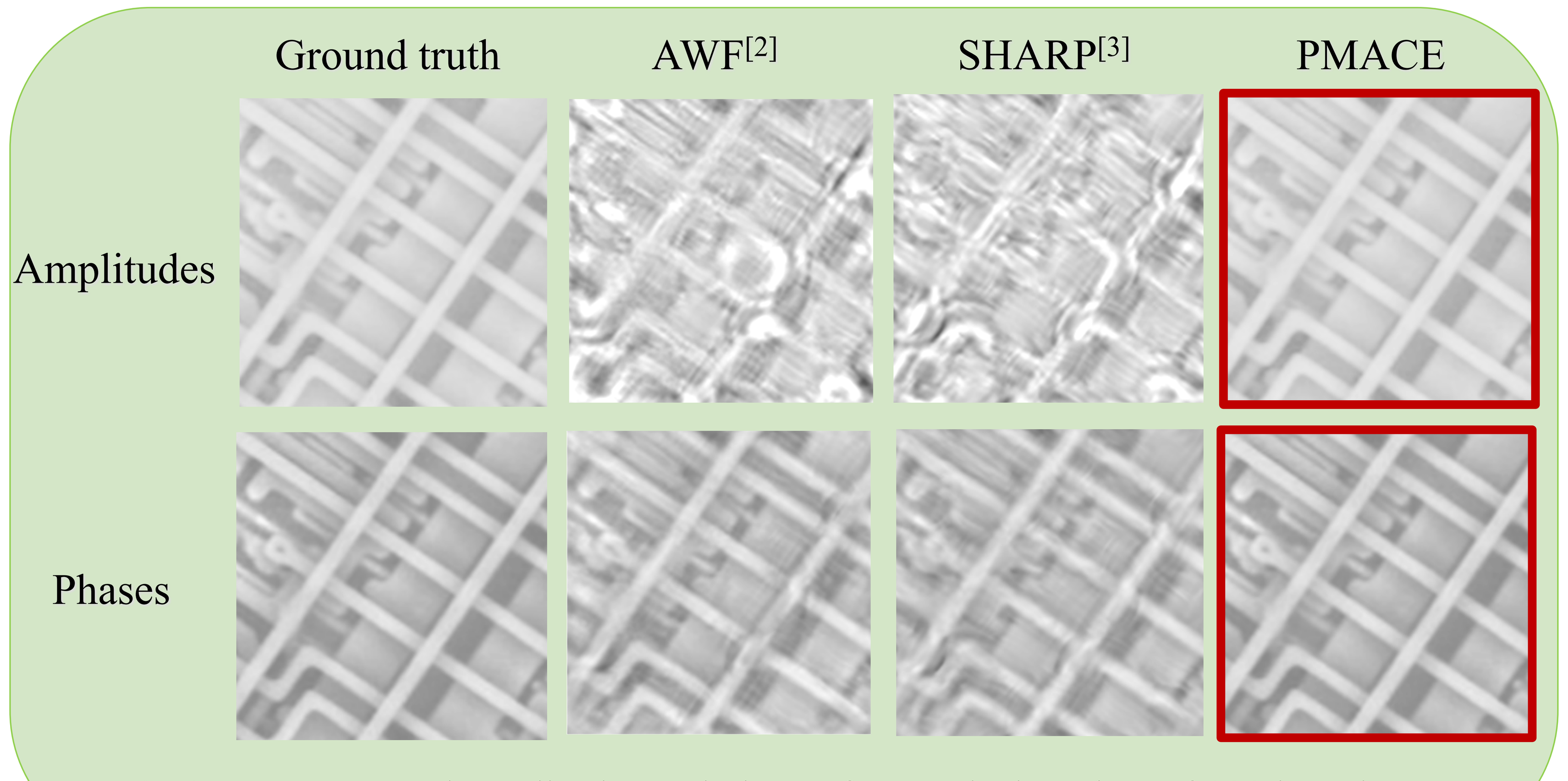


Fig 1. Reconstructed amplitudes and phases from noiseless data of 100 iterations.

- PMACE achieves faster convergence speed than competing algorithms.

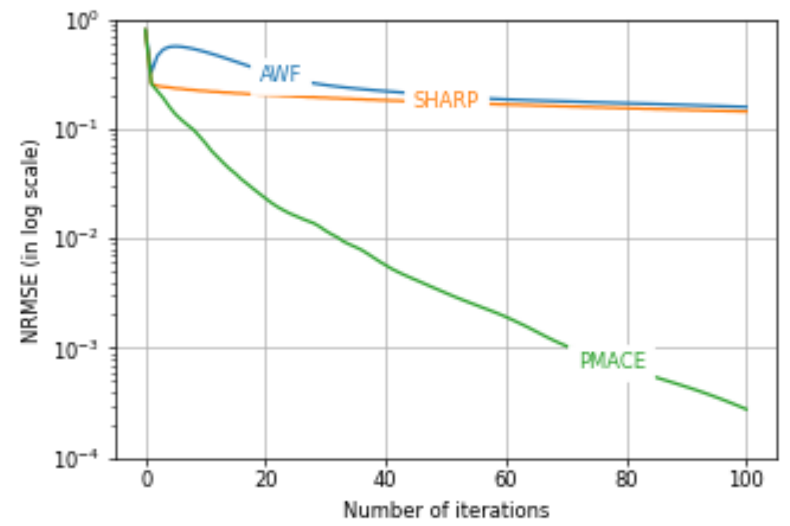


Fig 2. Convergence plots for reconstruction on noiseless data

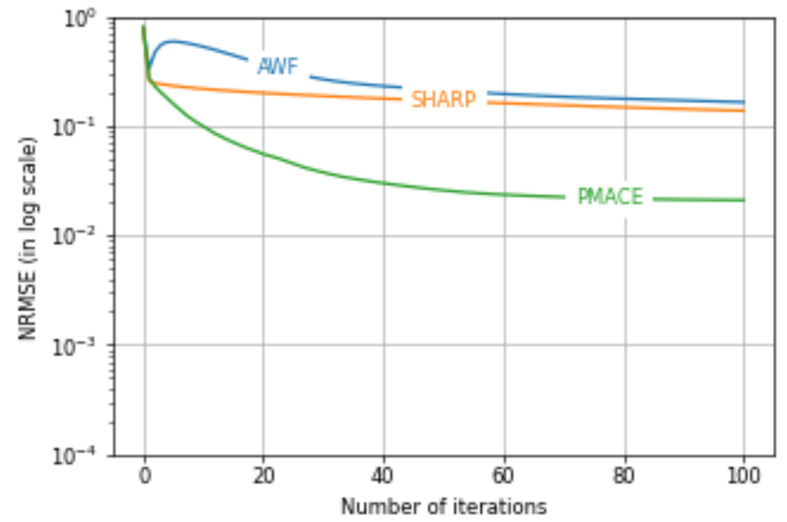


Fig 3. Convergence plots for reconstruction on noisy data

Conclusion

We proposed the PMACE approach for ptychographic image reconstruction. The proposed approach is easily parallelized and its convergence is guaranteed under appropriate hypotheses. Our results indicate that PMACE outperforms competing algorithms in terms of both convergence speed and reconstruction quality.

Acknowledgement

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Reference

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