Correction of Gain Fluctuations in Iterative Tomographic Image Reconstruction

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***Abstract*—Iterative reconstruction (IR) of x-ray computed tomographic (CT) images replaces single-pass computation with recursive refinement of the match between measured data and simulated forward projection of a candidate reconstruction. Whereas conventional filtered backprojection (FBP) includes a ramp filter which suppresses the DC component of the sinogram data, iterative methods seek a reconstruction which matches all components. Incorrect gains used in normalizing the sinogram projections for tube signal strength frequently occur with truncated scans of larger, low attenuation objects, and may produce artifacts in IR. A joint estimation of the true gain parameters and the image greatly reduces the artifacts in typical clinical CT imagery.**

***Index Terms*—Computed tomography, sensor gain correction, iterative methods, maximum *a posteriori* estimation.**

I. Introduction

Iterative methods of CT image reconstruction show promise in improving noise suppression and enhancing resolution in applications where limiting radiation exposure is essential, or data are limited. During the past decade, these algorithms have advanced significantly toward clinical applicability [1]–[4]. A common practical problem is the truncation of projections to the scanner's fan beam, which may occur with a large patient, poor placement of the patient, or the presence of blankets and other attenuating materials which fall outside the beam for some angles, and may even rest on the scanner gantry cover. An advantage of iterative reconstruction (IR) is that such image content may be estimated from limited angle information.

Raw data from the x-ray detector array typically undergo a number of corrections before reconstruction, regardless of the algorithm to be applied. Among these is reference normalization, which addresses the impact of fluctuations in the x-ray tube current output on the projections. For this purpose, a set of reference channels is placed slightly outside the scan field-of-view to measure the x-ray photons directly from the x-ray tube without attenuation by the scanned object. Coefficients calculated from these channels monitor the x-ray flux, and are used to normalize the projections relative to one another [5]. When an object is present outside the scan field-of-view, however, the reference channels are at least partially blocked, and pre-processing may not accurately compute the correction coefficients. This may result in inaccurate projection measurements generating image artifacts.

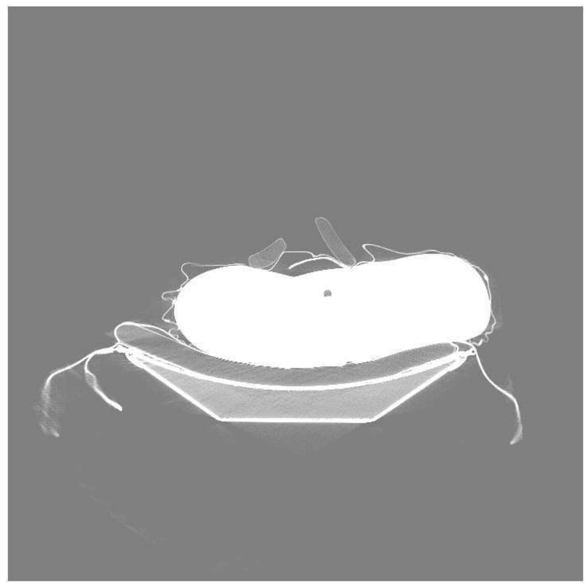

Fig. 1. CT reconstruction of a patient scan in 70cm field of view, with display window centered at 1000 Hounsfield units (HU), and of width 400 HU.

The image in Figure 1 illustrates a patient lying atop a plastic membrane, or slicker, that is intended to protect the scanner from fluids that could otherwise enter the mechanism of the CT table. In this case, the slicker falls well outside the cone of the scanner and rests on the gantry cover on both sides. It therefore interferes with the reference channel measurements for multiple projection angles.

Complete blockage of the reference channels is easily detected but partial blockage, as in the case above with interference by low-attenuation material, is less obvious. The result of this attenuation of normalization rays is spurious oscillation in the multiplicative factor applied to each view's received intensity of radiation. Figure 2 shows a temporally varying x-ray tube signal strength whose normalization would be expected to be proportional to the inverse of the upper signal. The lower plot shows the received normalization factors as a function of view angle. The approximately periodic oscillations in the normalization signal correspond to intermittent

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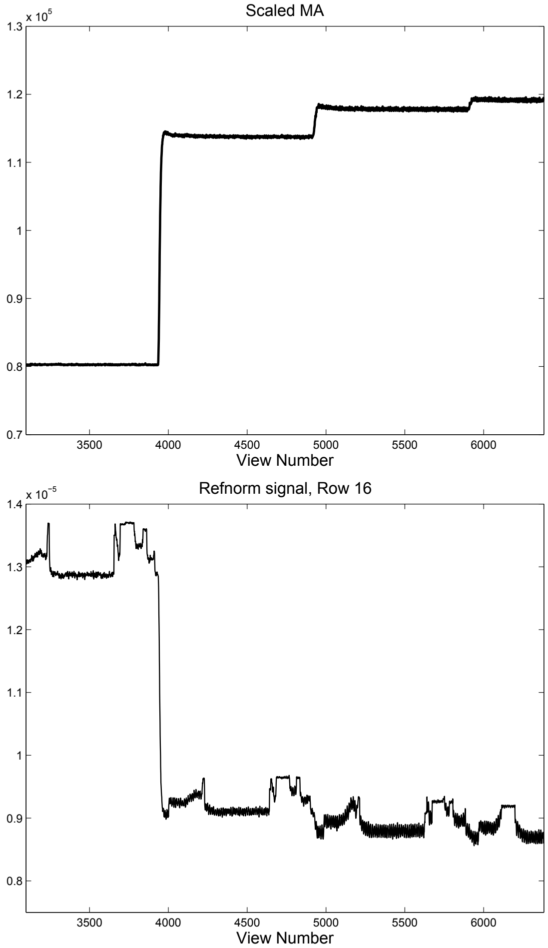

Fig. 2. X ray tube signal strength (above) as a function of view angle; reference normalization values (below).

partial blockage of the normalization channels by the slicker pictured in Figure 1.

This offset in the multiplicative factor $\ _i$ for correction of x-ray intensity values λ_i translates into approximately additive errors in the estimated integral attenuation values y_i, as

$$y_i \approx \ln\left(\frac{\lambda_T}{\ _i \lambda_i}\right) \tag{1}$$

if the tube signal strength in conversion is modeled as the constant λ_T.

Reconstructions from FBP have the contribution of any constant-valued (DC) component suppressed by the high-frequency emphasis ramp filter. Though rebinning may transform the fan-beam view-by-view bias into a more general low-frequency error, it is still heavily attenuated by the ramp provided gain fluctuations are not very high frequency in the view angle (and temporal) variable. The effects of gain fluctuation are evident in Figure 3, where low frequency shading is problematic in the IR image, but negligible in the FBP image. It is possible to replace the detector-based reference normalization with simply the inverse of the tube strength settings, but such a change would sacrifice valuable calibration

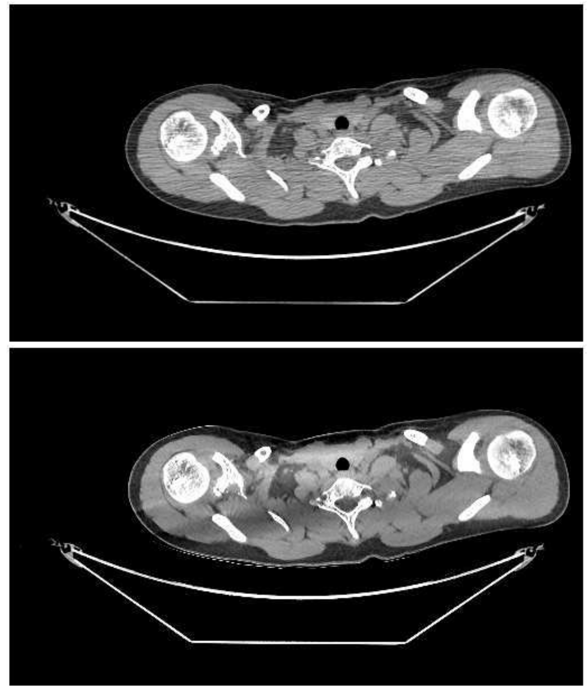

Fig. 3. FBP reconstruction of patient shoulder scan (above) and IR version of the same slice without correction for gain fluctuations (below). The display window width is 400 HU.

information concerning tube output variation relative to input power.

II. Gain Factor Estimation

A. Statistical Modeling

The log-likelihood function, parameterized in integral projection values, can be usefully approximated by a quadratic of the form

$$\ln p(y|\beta, x) \approx \quad \frac{1}{2}(y \quad \beta \quad Ax)^T W (y \quad \beta \quad Ax) + c(y) \tag{2}$$

in which y is the projection data, x is the unknown 3D image, A is the forward projection operator, W is a weighting matrix with entries proportional to the received radiation strength, $c(y)$ is constant in the parameter vector x, and β is the vector of transformed gain factors from $\beta_i = \ln \ _i$. The β_i are assumed fixed for any particular row and view, which guarantees that the number of parameters to be estimated for gain correction is small relative to the number of voxels in the 3D image. This constraint allows the complete set of sinogram corrections to be written as $\beta = Bf$ for a $M \quad L$ matrix B with $M >> L$ and all entries 0 or 1.

We pursue a maximum *a posteriori* probability (MAP) reconstruction with a simple *a priori* model for x:

$$\ln g(x) = \quad \sum_{j,k} b_{j,k} \quad (x_j \quad x_k) + \text{const.} \tag{3}$$

The function $\ (\,)$ is strictly convex. For the results in this paper, we employ:

$$(\delta) = \frac{|\delta|^p}{1 + \left|\frac{\delta}{c}\right|^{p \quad q}} \tag{4}$$

with $p = 2.0$ and $q = 1.2$, which approximates a quadratic penalty for small local differences and less rapidly increasing penalties for large differences. As the estimator of the image x, we seek:

$$(\hat{x}, \hat{f}) = \arg\max_{x,f} \{\ln p(y|f, x) + \ln g(x)\}.$$

This may be interpreted as the MAP estimate of the parameters (x, f) with a non-informative prior on f. The variation of the gains in f may also be assigned an *a priori* model, but the results in this paper do not include it.

B. Computation of the Estimate

The introduction of gain correction into the IR process has very little effect on computational cost. The least-squares optimization for the correction of each row and view involves only a number of variables equal to the number of detector channels. Suppose the detectors for a given row and view are indexed consecutively from m to n and receive the k-th gain factor in f. Let the vector e represent the differences between sinogram and forward projection values in the current state, $y \quad Bf \quad Ax$. The update of the f_k for these data is:

$$\Delta_{f_k} = \frac{\sum_{j=m}^{n} w_j e_j}{\sum_{j=m}^{n} w_j}, \tag{5}$$

the weighted average of entries in e. We have suppressed indices of the iteration step for simplicity.

Our overall optimization approach follows the sequential pattern used in our previous IR, iterative coordinate descent (ICD), with the image vector augmented by the gain parameters of f. We use spatially non-homogeneous ICD (NH-ICD), which mixes full passes among active voxels with updates according to voxels' history [6]. During the latter of these two phases, those voxels with the largest last update magnitudes are visited first. A full, sequential pass through f, optimizing with respect to the shift in each view and row, follows a number of voxel updates equivalent to a pass through approximately 1/4th of the image x.

ICD has demonstrated satisfactory convergence in many reconstruction problems in fewer than 10 iterations when initiated with the FBP reconstruction. The forward projection of the FBP image yields a relatively accurate, consistent set of low-frequency components to compare with the sinogram data. This allows an initial correction for most of the error in gain. In Figure 4, we see that the estimates of f converge quite rapidly, showing little movement after 5 update steps, which corresponds to less than two iterations of NH-ICD.

To establish the global convergence of the iterates, we may simply augment the vector x with all the $\{f_k\}$ to form $\tilde{x}$ and add the same number of columns to A to form $\tilde{A}$ with unity in its columns capturing the proper element of f for each datum. The modified, approximate log-likelihood function then appears as

$$\ln p(y|\tilde{x}) \approx \quad \frac{1}{2}(y \quad \tilde{A}\tilde{x})^T W (y \quad \tilde{A}\tilde{x}),$$

that is in the same form as in previous incarnations without gain correction. Assuming that the null space of $\tilde{A}$ is empty,

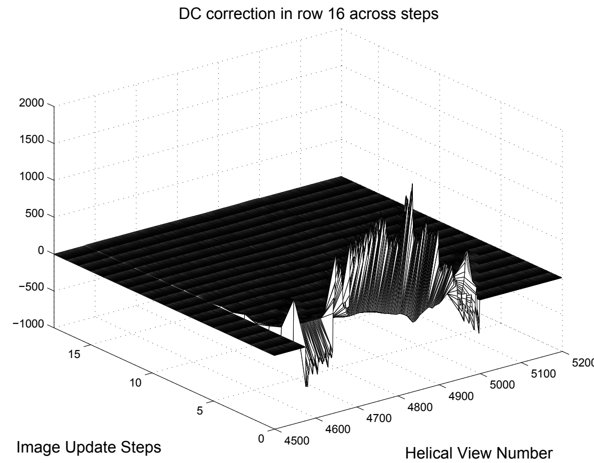

Fig. 4. Estimates of residual constant offset, resulting from gain error, in row 16 of a 32 row scan, estimated after successive image updates. Sums along update steps for fixed view yield the final estimate of β_i's for row 16 and corresponding view.

the complete objective function is strictly concave in $\tilde{x}$. The second derivatives of the MAP cost function in all voxel values are bounded above by the log prior, and the second derivative with respect to f_k is equal to $\quad \sum_{j=m}^{n} w_j$, using the same indices for computing the k-th correction factor as in (5). Given this formulation for the MAP objective function, the ICD iterates applied to all elements of $\tilde{x}$ are guaranteed globally, monotonically convergent by the results of [7].

C. Results of Gain Correction

Because the gain parameters are relatively few, their estimation is robust and convergence is rapid. The shading artifacts resulting from inaccurate normalization are removed from the IR images of the patient shoulder scan considered above, as shown in Figure 5. The joint image/gain correction estimate shows the advantages in noise suppression more typical of IR reconstructions.

Although this method may be thought of as a removal of the DC component from the error state vector for iterative reconstruction, it is important to emphasize that we are performing estimation of gain *in addition to* image content, and nothing in this process prevents improvement in the DC content of the image from its initial state. Without special processing, truncated projections necessarily lead to FBP reconstructions with mass in projection data which is unrecovered in the image, and induce significant artifacts at the edge of the scan field-of-view where truncation occurs. Estimation of this truncated content is crucial for high-quality iterative image estimates, where the reconstruction of all sources of measured x-ray attenuation is necessary to ensure consistency between the IR image and the projection data.

To demonstrate the capability of iterative methods with gain correction to recover truncated material from the FBP initial reconstruction, we have scanned a phantom protruding beyond the fan of the scanner beams, with the FBP result shown in Figure 6 (top). This reconstruction serves as the initial state

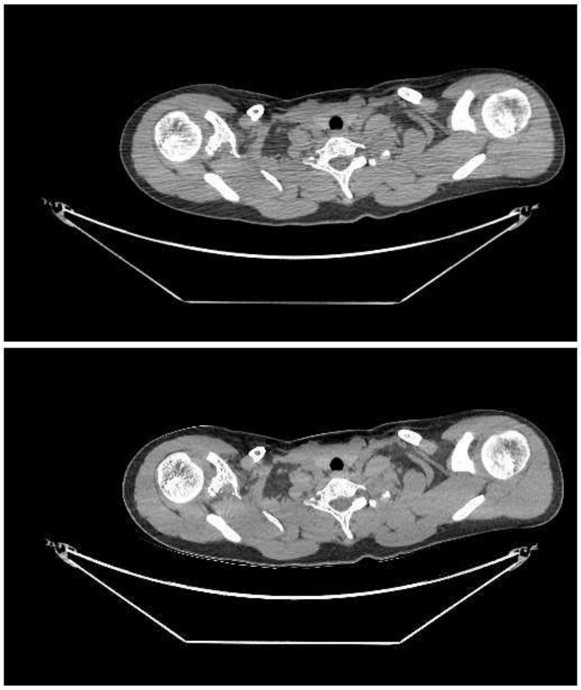

Fig. 5. FBP reconstruction with standard data (above) and IR reconstruction with gain fluctuation correction (below). The display window width is 400 HU.

for MAP-ICD estimation with the modified log-likelihood for gain correction. Because reference channel blockage is detected when the phantom intervenes, gain fluctuations are not a serious problem here. The IR results, first without, then with the gain correction, are shown in the lower figures (center and bottom, respectively). While the reconstruction of the edge of the phantom may be imperfect, it does show useful recovery of image information outside the normal field of view. The gain correction does not prohibit the proper estimation of the truncated regions of the scanned object, but rather allows some improvement in the uniformity of the result relative to the IR image without gain correction.

III. Conclusion

Preliminary results suggest that we can simply, effectively remove the artifacts due to spurious gain fluctuation in sinogram data by using the iterative reconstruction process to estimate the additive sinogram errors due to multiplicative error at the detectors. The joint estimation of the gains with other image content does not negatively affect reconstruction from truncated projections. Further research will test the effectiveness of the method on an ensemble of clinical data sets.

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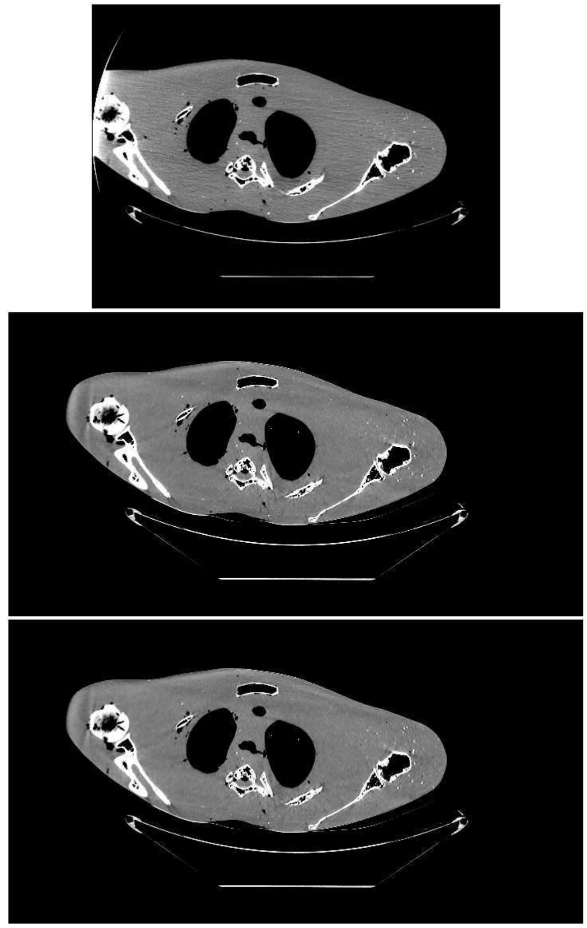

Fig. 6. FBP reconstruction from truncated data of a shoulder phantom (top) and IR reconstructions without (center) and with (bottom) gain fluctuation correction. The FBP image is reconstructed with a diameter of 50cm (the size of the scan field of view), whereas the IR image is reconstructed with a diameter of 70cm (the size of the full bore of the CT scanner) to include all possible sources of x ray attenuation. The display window width is 400 HU.

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