Digital Halftoning

- Many image rendering technologies only have binary output. For example, printers can either “fire a dot” or not.
- Halftoning is a method for creating the illusion of continuous tone output with a binary device.
- Effective digital halftoning can substantially improve the quality of rendered images at minimal cost.

Thresholding

- Assume that the image falls in the range of 0 to 255.
- Apply a space varying threshold, $T(i,j)$.

$$b(i,j) = \begin{cases} 255 & \text{if } X(i,j) > T(i,j) \\ 0 & \text{otherwise} \end{cases} .$$

- What is $X(i,j)$?
- Lightness
  - Larger $\Rightarrow$ lighter
  - Used for display
- Absorptance
  - Larger $\Rightarrow$ darker
  - Used for printing
- $X(i,j)$ will generally be in units of absorptance.

Constant Threshold

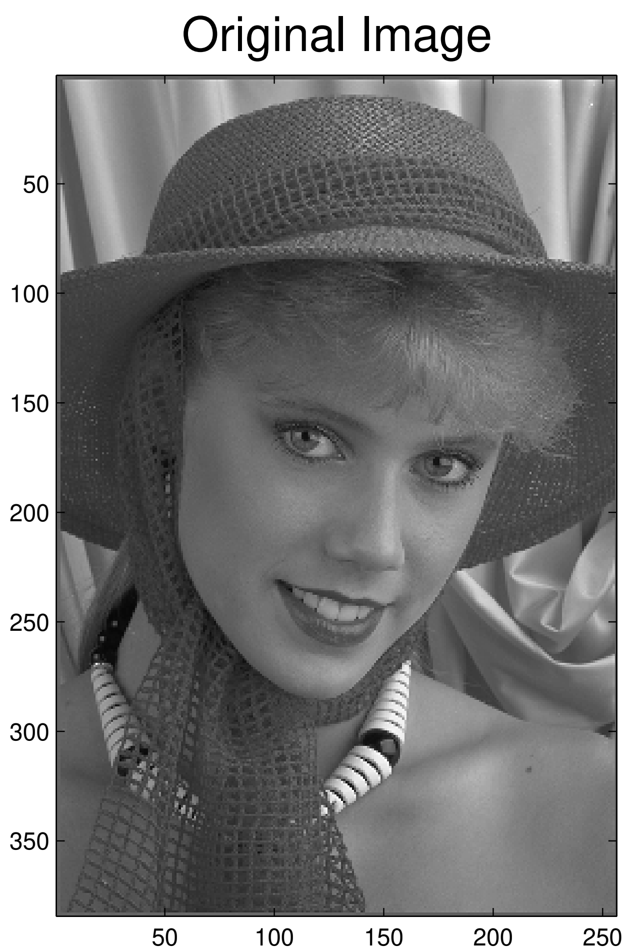
- Assume that the image falls in the range of 0 to 255.
- $255 \Rightarrow Black$ and $0 \Rightarrow White$
- The minimum squared error quantizer is a simple threshold

$$b(i,j) = \begin{cases} 255 & \text{if } X(i,j) > T \\ 0 & \text{otherwise} \end{cases} .$$

  where $T = 127$.
- This produces a poor quality rendering of a continuous tone image.

The Minimum Squared Error Solution

- Threshold each pixel
  - Pixel> 127 Fire ink
  - Pixel≤ 127 do nothing

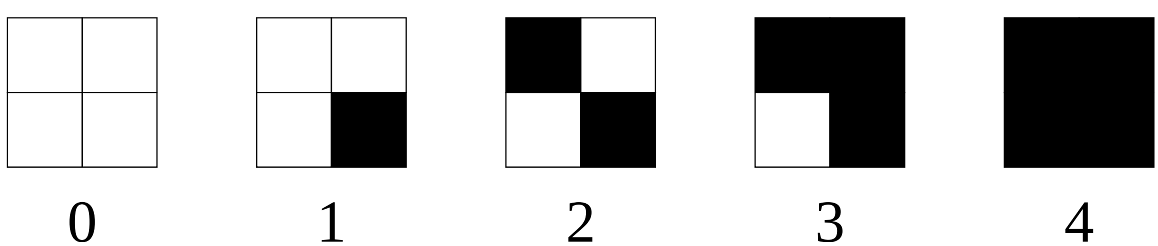


Ordered Dither

- For a constant gray level patch, turn the pixel "on"in a specified order.
- This creates the perception of continuous variations of gray.
- An $N \times N$ index matrix specifies what order to use.

$$I_2(i,j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

- Pixels are turned on in the following order.



Implementation of Ordered Dither via Thresholding

- The index matrix can be converted to a "threshold matrix" or "screen" using the following operation.

$$T(i,j) = 255\frac{I(i,j)+0.5}{N^2}$$

- The $N \times N$ matrix can then be "tiled" over the image using periodic replication.

$$T(i \,\mathrm{mod} N, j \,\mathrm{mod} N)$$

- The ordered dither algorithm is then applied via thresholding.

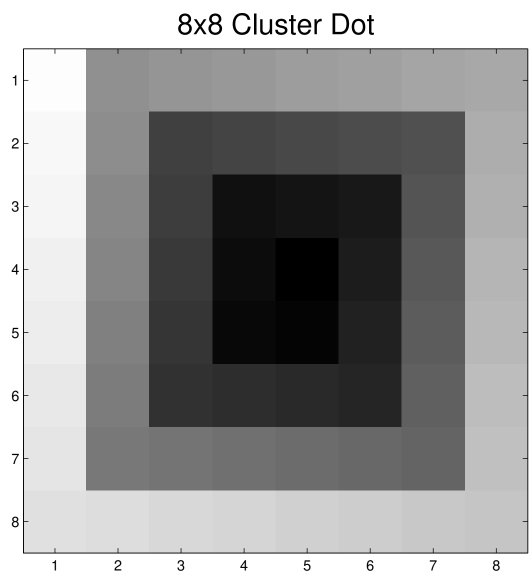
$$b(i,j) = \begin{cases} 255 & \text{if } X(i,j) > T(i \,\mathrm{mod} N, j \,\mathrm{mod} N) \\ 0 & \text{otherwise} \end{cases} .$$

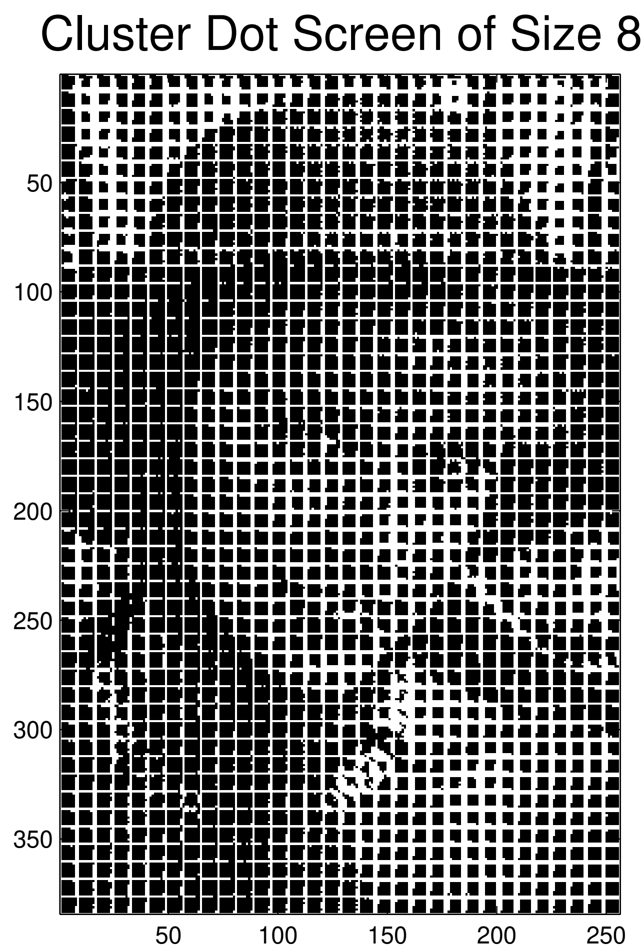
Clustered Dot Screens

- Definition: If the consecutive thresholds are located in spatial proximity, then this is called a “clustered dot screen.
- Example for 8×8 matrix:

62	57	48	36	37	49	58	63
56	47	35	21	22	38	50	59
46	34	20	10	11	23	39	51
33	19	9	3	0	4	12	24
32	18	8	2	1	5	13	25
45	31	17	7	6	14	26	40
55	44	30	16	15	27	41	52
61	54	43	29	28	42	53	60

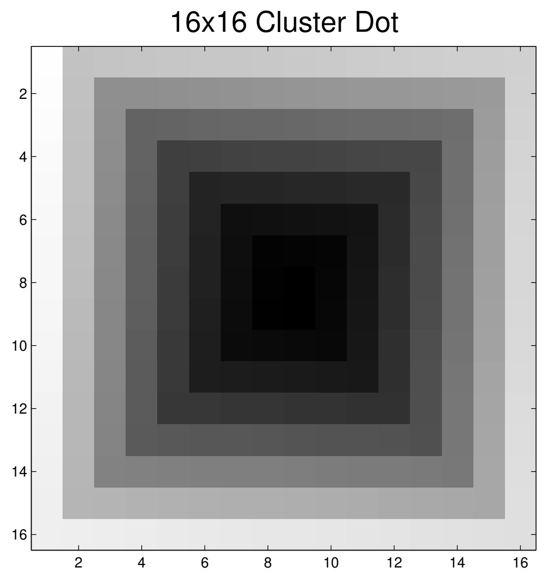
Example: 8×8 Clustered Dot Screening

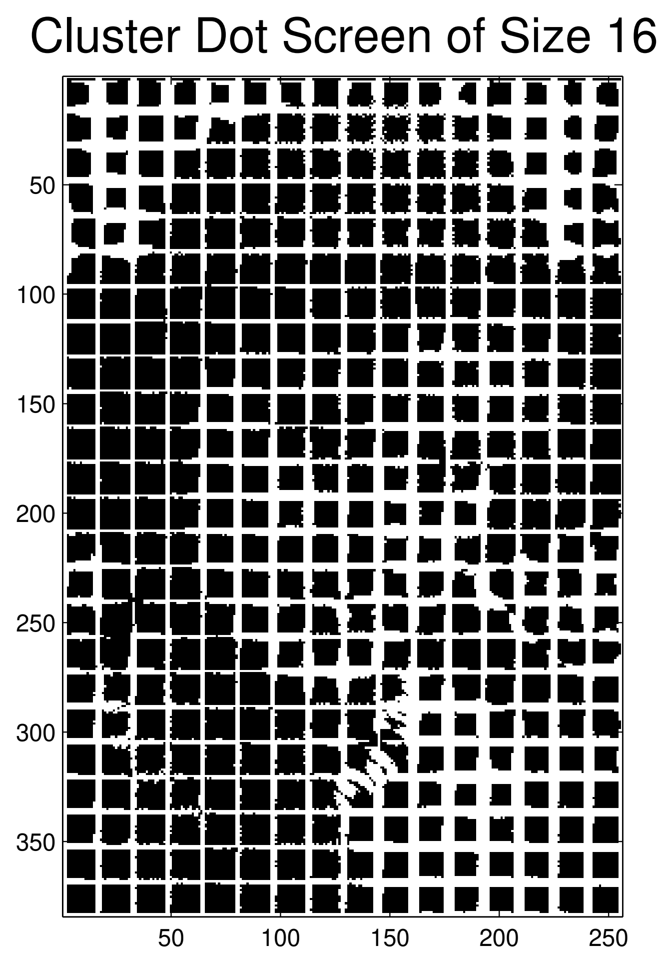




- Only supports 65 gray levels.

Example: 16×16 Clustered Dot Screening





- Support a full 257 gray levels, but has half the resolution.

Properties of Clustered Dot Screens

- Requires a trade-off between number of gray levels and resolution.
- Relatively visible texture
- Relatively poor detail rendition
- Uniform texture across entire gray scale.
- Robust performance with non-ideal output devices
  - Non-additive spot overlap
  - Spot-to-spot variability
  - Noise

Dispersed Dot Screens

- Bayer's optimum index Matrix (1973) can be defined recursively.

$$I_2(i,j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$I_{2n} = \begin{bmatrix} 4 * I_n + 1 & 4 * I_n + 2 \\ 4 * I_n + 3 & 4 * I_n \end{bmatrix}$$

- Examples

2×2

1	2
3	0

4×4

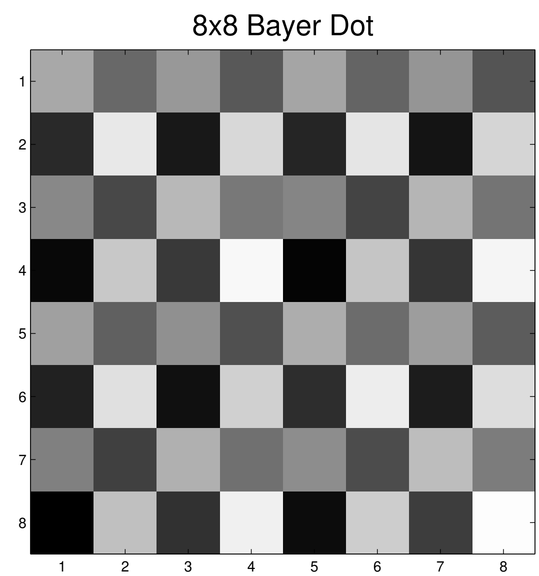
5	9	6	10
13	1	14	2
7	11	4	8
15	3	12	0

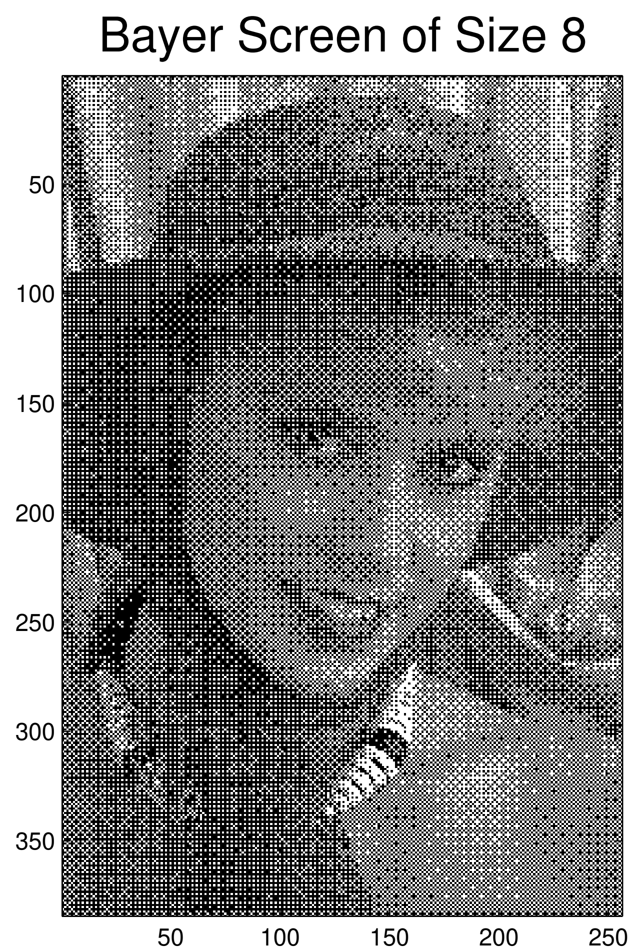
8×8

21	37	25	41	22	38	26	42
53	5	57	9	54	6	58	10
29	45	17	33	30	46	18	34
61	13	49	1	62	14	50	2
23	39	27	43	20	36	24	40
55	7	59	11	52	4	56	8
31	47	19	35	28	44	16	32
63	15	51	3	60	12	48	0

- Yields finer amplitude quantization over larger area.
- Retains good detail rendition within smaller area.

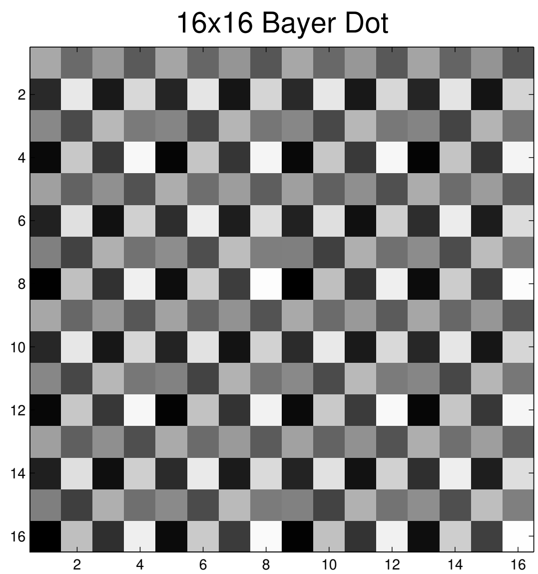
Example: 8×8 Bayer Dot Screening

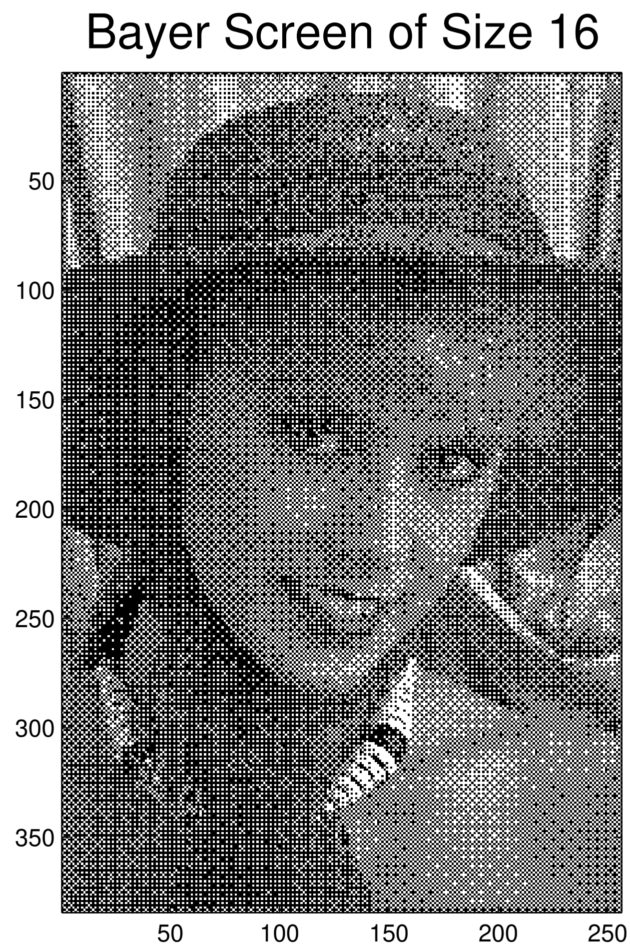




- Again, only 65 gray levels.

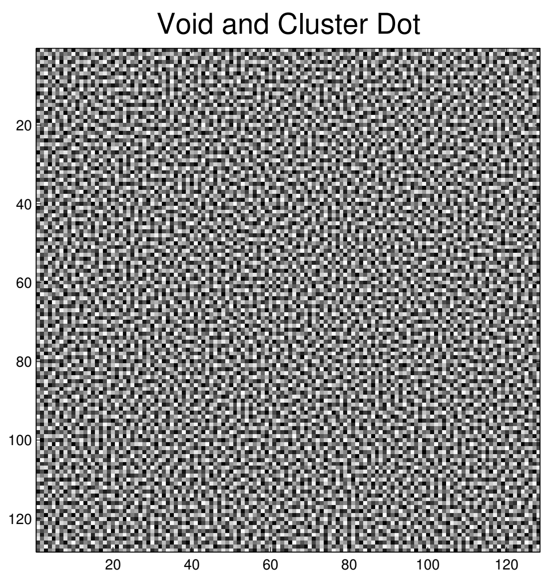
Example: 16×16 Bayer Dot Screening

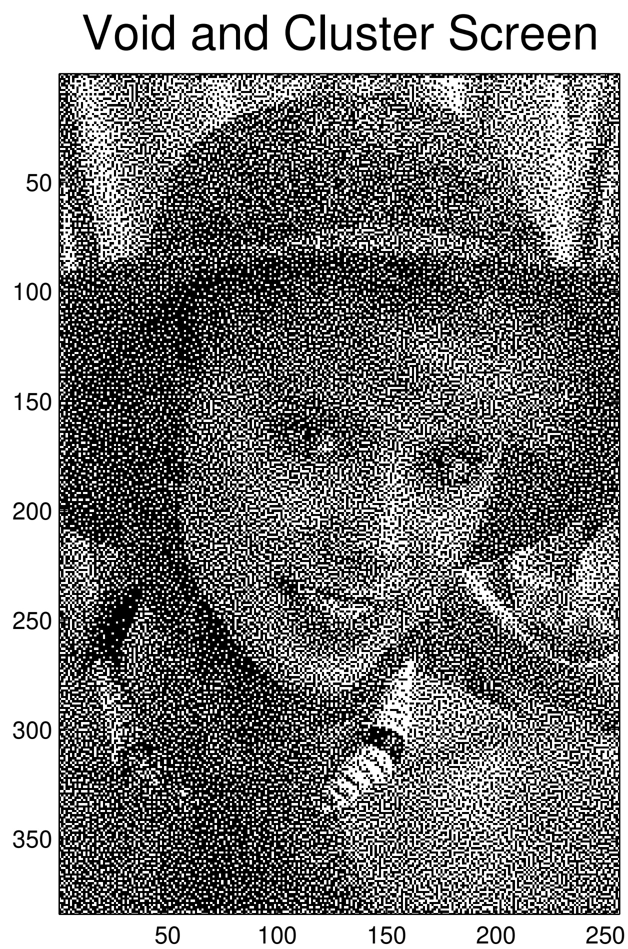




- Doesn't look much different than the 8×8 case.
- No trade-off between resolution and number of gray levels.

Example: 128×128 Void and Cluster Screen (1989)





- Substantially improved quality over Bayer screen.

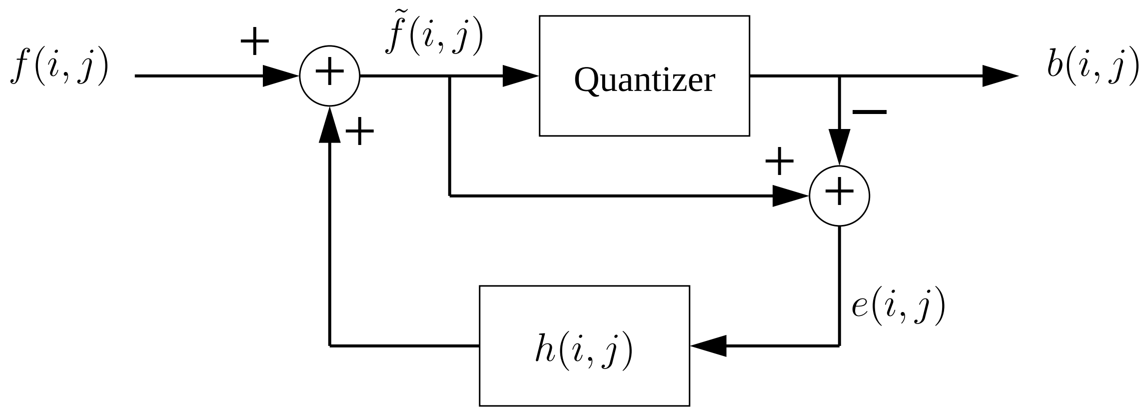
Properties of Dispersed Dot Screens

- Eliminate the trade-off between number of gray levels and resolution.
- Within any region containing K dots, the K thresholds should be distributed as uniformly as possible.
- Textures used to represent individual gray levels have low visibility.
- Improved detail rendition.
- Transitions between textures corresponding to different gray levels may be more visible.
- Not robust to non-ideal output devices
  - Requires stable formation of isolated single dots.

Error Diffusion

- Error Diffusion
  - Quantizes each pixel using a neighborhood operation, rather than a simple pointwise operation.
  - Moves through image in raster order, quantizing the result, and “pushing” the error forward.
  - Can produce better quality images than is possible with screens.

Filter View of Error Diffusion



- Equations are

$$b(i,j) = \begin{cases} 255 & \text{if } \tilde{f}(i,j) > T \\ 0 & \text{otherwise} \end{cases}$$

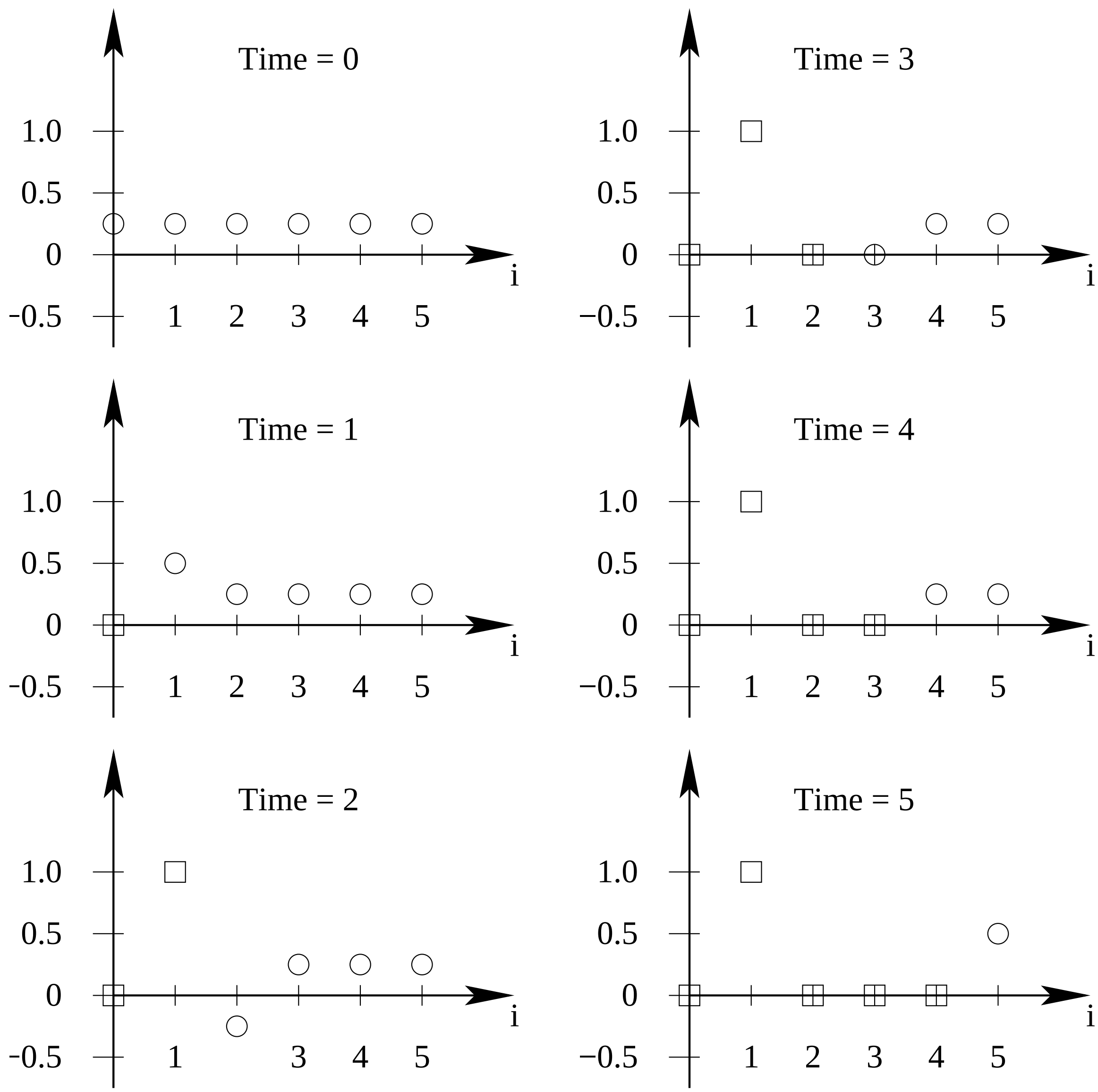
$$e(i,j) = \tilde{f}(i,j) - b(i,j)$$

$$\tilde{f}(i,j) = f(i,j) + \sum_{k,l \in S} h(k,l) e(i-k, j-l)$$

- Parameters
  - Threshold is typically $T = 127$.
  - $h(k,l)$ are typically chosen to be positive and sum to 1

1-D Error Diffusion Example

- $\tilde{f}(i) \Rightarrow$ circles
- $b(i) \Rightarrow$ boxes



Two Views of Error Diffusion

- Two mathematically equivalent views of error diffusion
  - Pulling errors forward
  - Pushing errors ahead
- Pulling errors forward
  - More similar to common view of IIR filter
  - Has advantages for analysis
- Pushing errors ahead
  - Original view of error diffusion
  - Can be more easily extended to important cases when weights area time/space varying

ED: Pulling Errors Forward

1. For each pixel in the image (in raster order)

   (a) Pull error forward

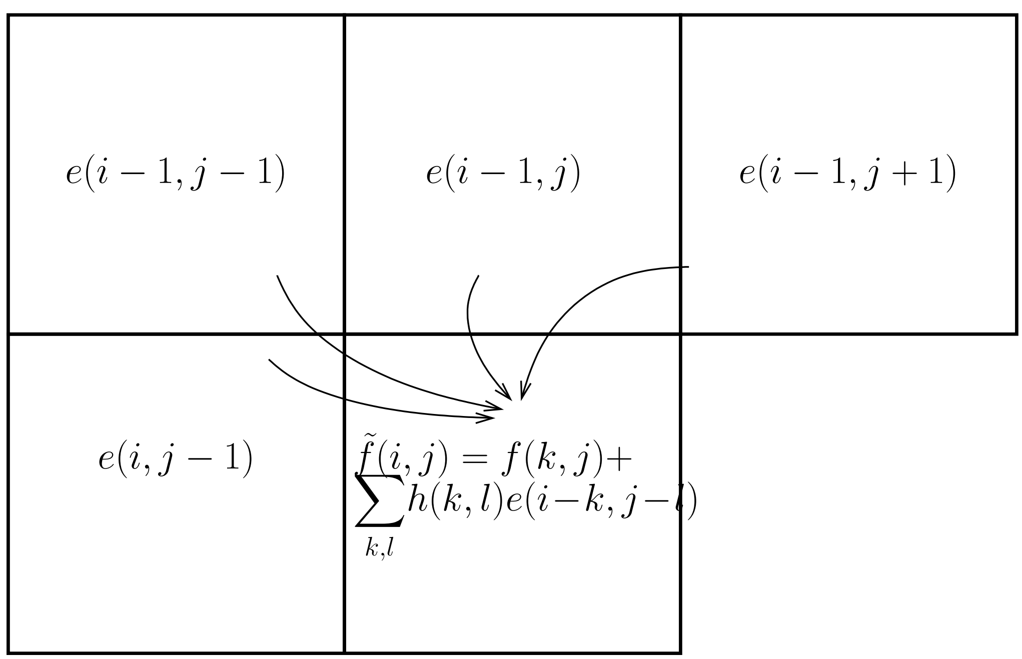
$$\tilde{f}(i,j) = f(i,j) + \sum_{k,l \in S} h(k,l)e(i-k,j-l)$$

   (b) Compute binary output

$$b(i,j) = \begin{cases} 255 & \text{if } \tilde{f}(i,j) > T \\ 0 & \text{otherwise} \end{cases}$$

   (c) Compute pixel's error

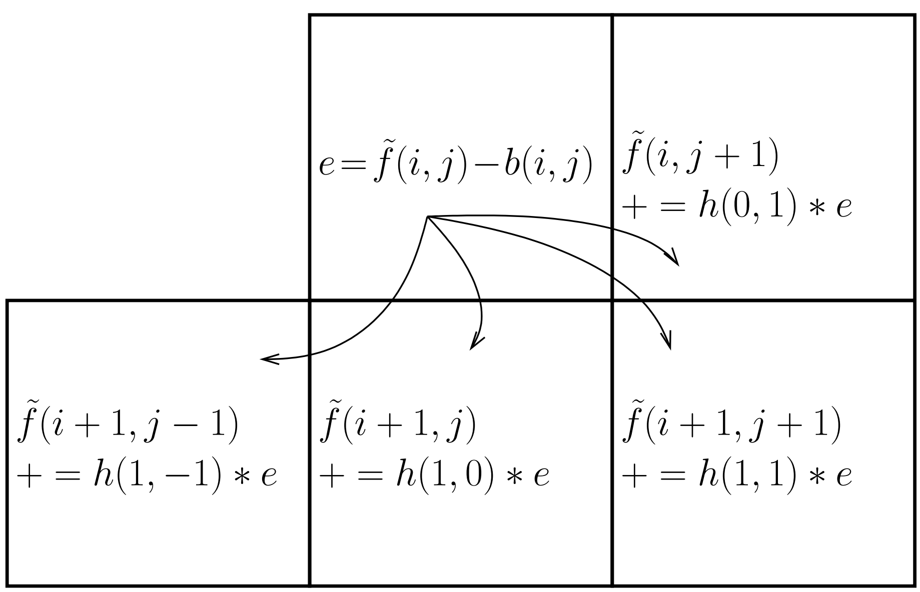
$$e(i,j) = \tilde{f}(i,j) - b(i,j)$$



2. Display binary image $b(i,j)$

ED: Pushing Errors Ahead

1. Initialize $\tilde{f}(i,j) \leftarrow f(i,j)$
2. For each pixel in the image (in raster order)
   (a) Compute
   $$b(i,j) = \begin{cases} 255 & \text{if } \tilde{f}(i,j) > T \\ 0 & \text{otherwise} \end{cases}$$
   (b) Diffuse error forward using the following scheme



3. Display binary image $b(i,j)$

Commonly Used Error Diffusion Weights

- Floyd and Steinberg (1976)

		7/16
3/16	5/16	1/16

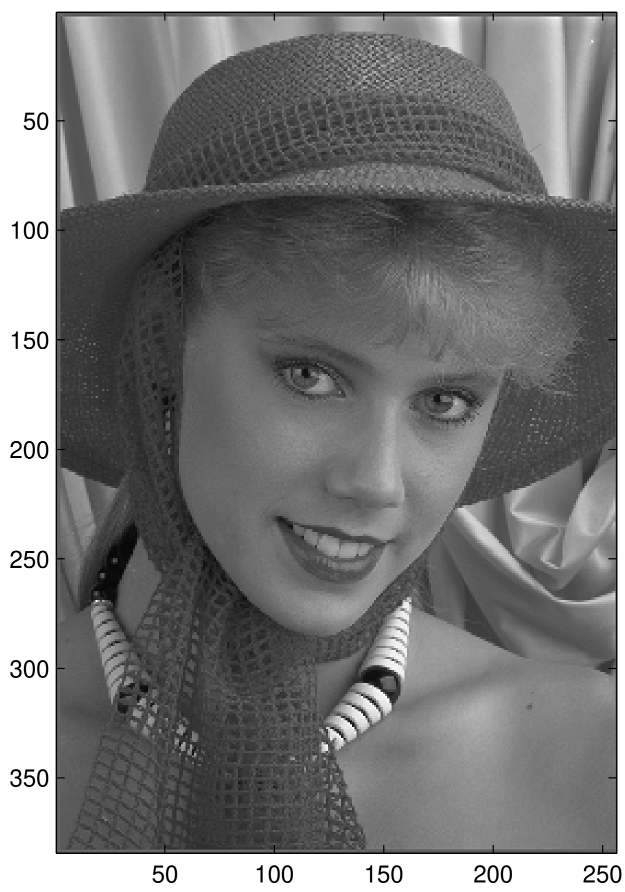
- Jarvis, Judice, and Ninke (1976)

			7/48	5/48
3/48	5/48	7/48	5/48	3/48
1/48	3/48	5/48	3/48	1/48

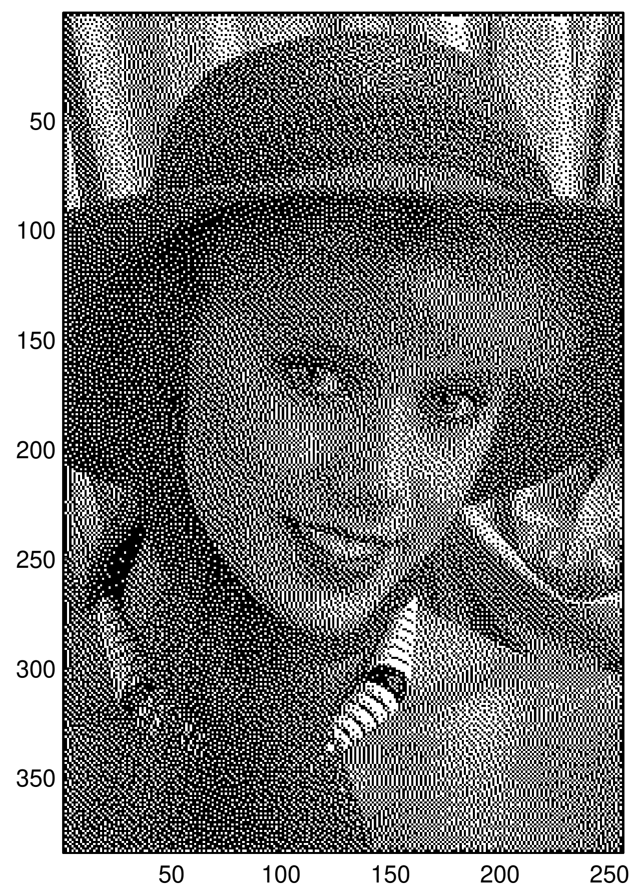
Floyd Steinberg Error Diffusion (1976)

- Process pixels in neighborhoods by “diffusing error” and quantizing.

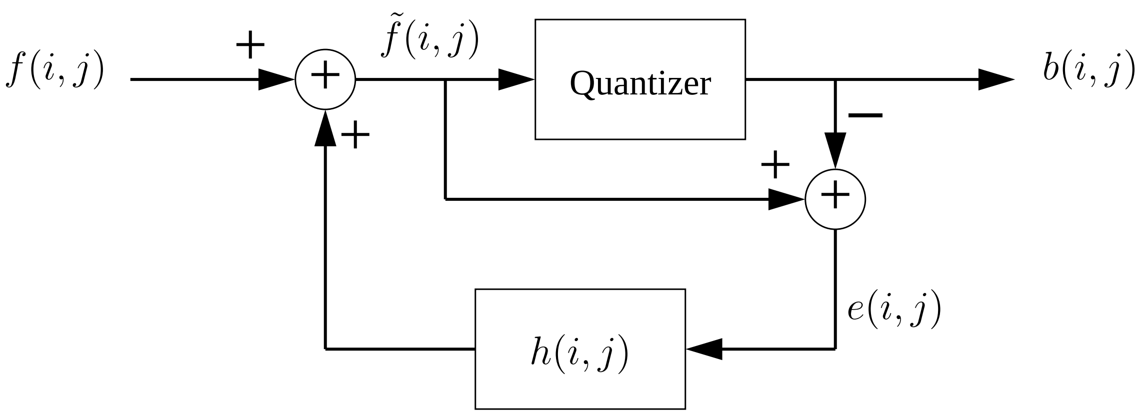
Original Image



Floyd and Steinberg Error Diffusion



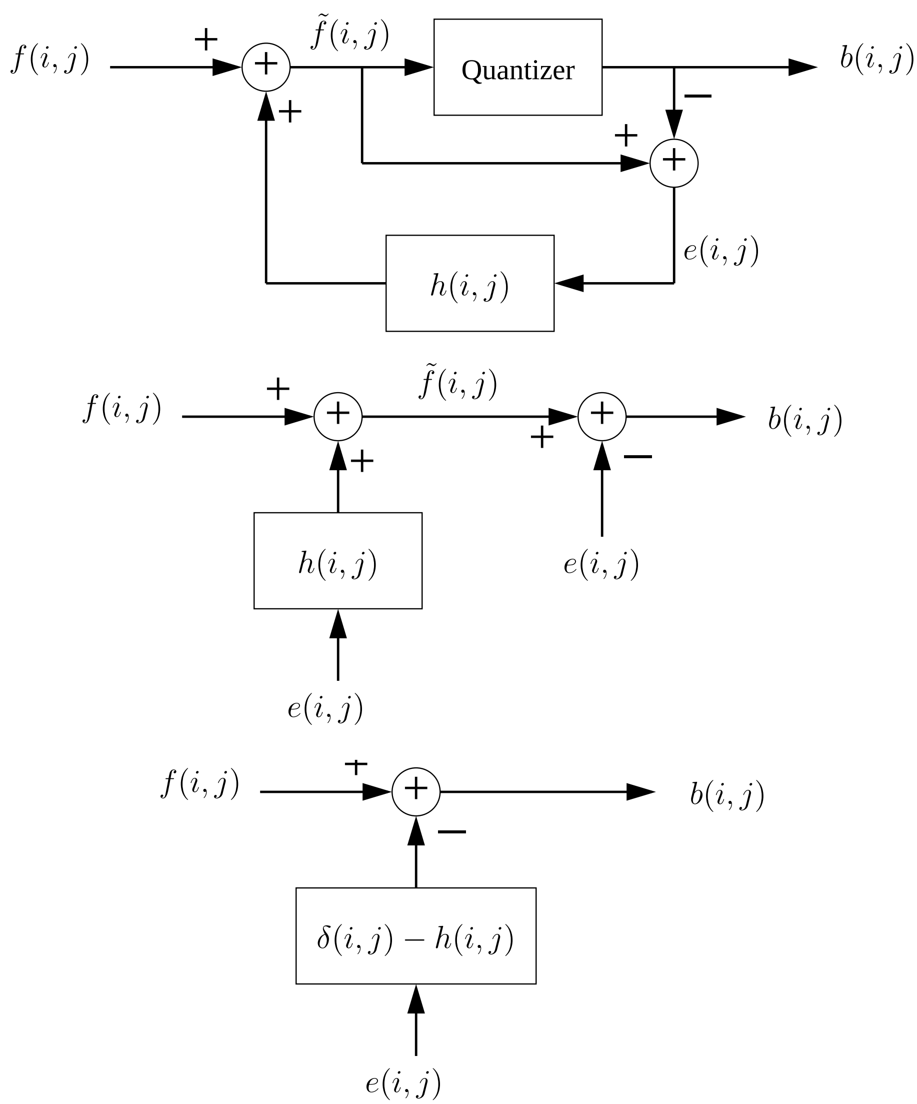
Quantization Error Modeling for Error Diffusion



- Quantization error is commonly assumed to be:
  - Uniformly distributed on $[-0.5, 0.5]$
  - Uncorrelated in space
  - Independent of signal $\tilde{f}(i,j)$
  - $E\left[e(i,j)\right] = 0$
  - $E\left[e(i,j)e(i+k,j+l)\right] = \frac{\delta(k,l)}{12}$

Modified Error Diffusion Block Diagram

- The error diffusion block diagram can be rearranged to facilitate error analysis



Error Diffusion Spectral Analysis

- So we see that

$$b(i,j) = f(i,j) - (\delta(i,j) - h(i,j)) * e(i,j)$$

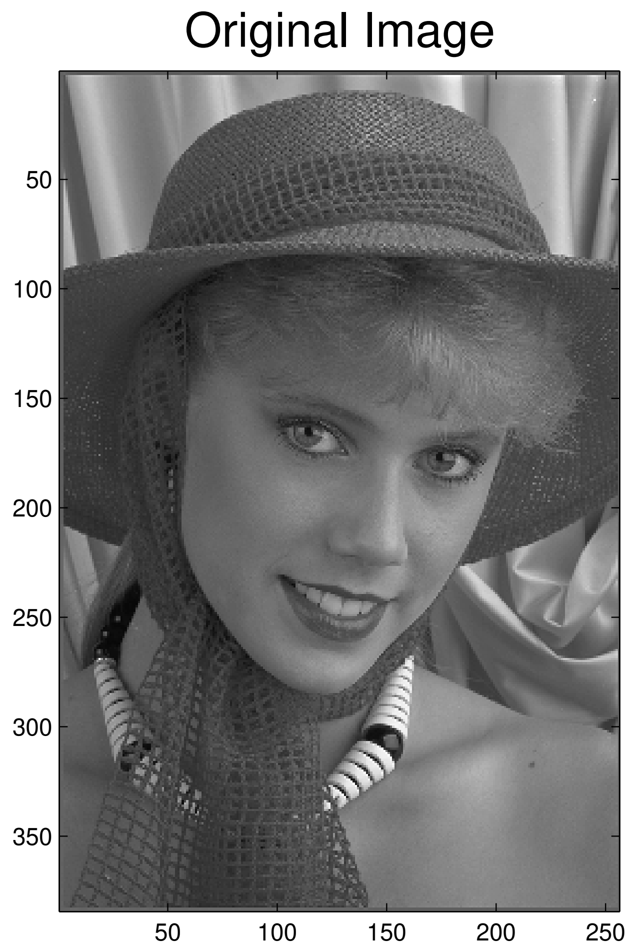
rewriting ...

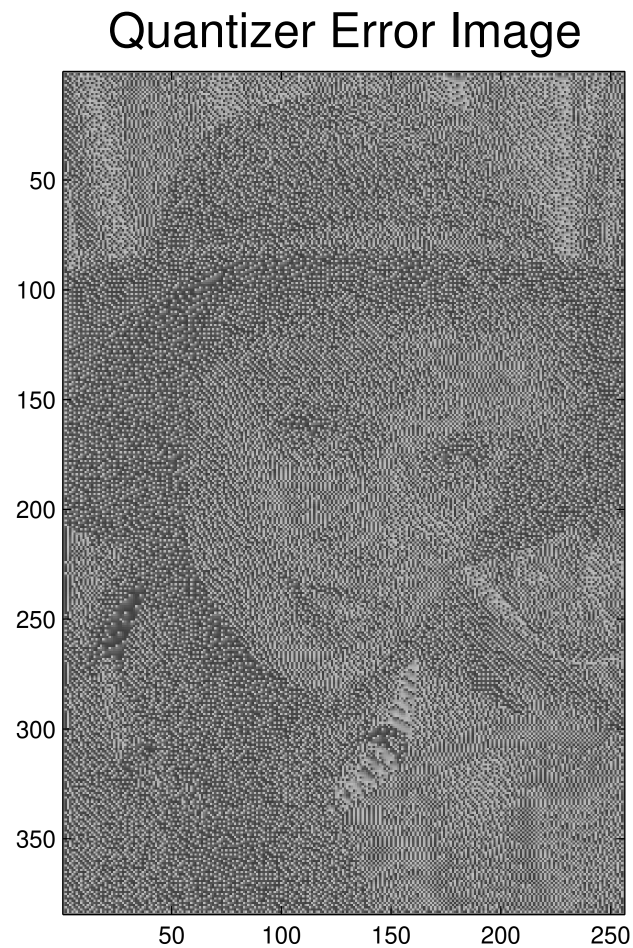
$$f(i,j) - b(i,j) = \underbrace{(\delta(i,j) - h(i,j))}_{\text{high pass filter}} * \underbrace{e(i,j)}_{\text{quantization error}}$$

  - Display error is $f(i,j) - b(i,j)$
  - Quantization error is $e(i,j)$
  - Display error is a high pass version of quantization error
  - Human visual system is less sensitive to high spatial frequencies

Error Image in Floyd Steinberg Error Diffusion

- Process pixels in neighborhoods by "diffusing error" and quantizing.





Correlation of Quantization Error and Image

- Quantizer error spectrum is unknown
- Quantizer error model

$$
\begin{aligned}
E(\mu,\nu) &= \rho F(\mu,\nu) + R(\mu,\nu) \\
&= \rho(\text{Image}) + (\text{Residual})
\end{aligned}
$$

  – ρ represents correlation between quantizer error and image

Weight	ρ
1-D	0.0
Floyd and Steinberg	0.55
Jarvis, Judice, and Ninke	0.8

- Using this model, we have

$$
\begin{aligned}
B(\mu,\nu) &= F(\mu,\nu) - (1 - H(\mu,\nu))\, E(\mu,\nu) \\
&= [1 - \rho\,(1 - H(\mu,\nu))]\, F(\mu,\nu) + \text{noise}
\end{aligned}
$$

- This is unsharp masking

Additional Topics

- Pattern Printing
- Dot Profiles
- Halftone quality metrics
  - Radially averaged power spectrum (RAPS)
  - Weighted least squares with HVS constrast sensitivity function
  - Blue noise dot patterns
- Error diffusion
  - Unsharp masking effects
  - Serpentine scan patterns
  - Threshold dithering
  - TDED
- Least squared halftoning
- Printing and display technologies
  - Electrophotographic
  - Inkjet