# Fast Mars Free-Returns via Venus Gravity Assist 

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Mars free-return trajectories that use a Venus flyby either before or after the Mars encounter are found, and an alternative launch opportunity for the Inspiration Mars mission is identified. Launch dates are searched from 2015 to 2060 , and focus is placed on identifying opportunities that have a short total TOF (i.e. that are "fast"), so that they may be used for a human mission to flyby Mars (similar to that proposed for Inspiration Mars). Constraints on Earth launch $V_{\infty}$ and Earth arrival $V_{\infty}$ are based on those used for the nominal Inspiration Mars opportunity in 2018. A set of near-term candidate trajectories are found using the gravity-assist path Earth-Venus-Mars-Earth. One such candidate, with launch date on November 22, 2021, has Earth launch and arrival $V_{\infty}$ of $4.50 \mathrm{~km} / \mathrm{s}$ and 6.53 $\mathrm{km} / \mathrm{s}$, respectively (both lower than the nominal Inspiration Mars trajectory), and with a total flight time of 582 days. Venus free-return opportunities are also found, with promising application for a human flyby mission to Venus.

## Nomenclature

| $h$ | Closest approach altitude at flyby, km |
| :--- | :--- |
| $V$ | Velocity, $\mathrm{km} / \mathrm{s}$ |
| $V_{\infty}$ | Hyperbolic excess velocity, $\mathrm{km} / \mathrm{s}$ |
| C 3 | Square of Earth launch $V_{\infty}\left(\mathrm{twice}\right.$ the specific hyperbolic energy) $\mathrm{km}^{2} / \mathrm{s}^{2}$ |
| $\Delta V$ | Impulsive change in velocity, $\mathrm{km} / \mathrm{s}$ |
| Subscripts |  |
| Arrival | Earth arrival |
| Entry | Entry into Earth's atmosphere (at 122 km altitude) |
| Launch | Earth launch |
| $M$ | Mars |
| $V$ | Venus |

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## I. Introduction

In the last several decades, there have been many trajectory designs for human missions to Mars. ${ }^{2-15,24}$ Despite this effort to show the way to Mars, humans have yet to stand on the surface of the red planet, and it remains a long-term goal of the US and several other nations.

Last year, Tito et al. ${ }^{13}$ proposed that a human Mars flyby mission, dubbed "Inspiration Mars," could be launched in 2018 with a two person (one man and one woman) crew. This proposal was based on a trajectory reported by Patel et al. ${ }^{10}$ who investigated Mars free-return trajectories with launch dates ranging from 1995 to 2020. In their paper, several notably "fast" trajectories were highlighted that had relatively short times of flight (TOF) of about 1.4 years. One of these fast trajectories, with a launch date on January 5, 2018, was selected by Tito et al. ${ }^{13}$ because its 501 day flight time, relatively low launch energy, and relatively low Earth entry speed were considered feasible with present day technology.

Unfortunately, a similar trajectory to the 2018 Mars free-return does not occur for another 15 years-an unacceptable delay for Inspiration Mars. In order to support the effort to fly the 2018 launch option, it is prudent to have an alternative launch opportunity in the near future. This paper will show that such an opportunity is available if Venus is employed as a gravity-assist body.

In 2002, Okutsu and Longuski ${ }^{11}$ investigated Mars free-return trajectories that incorporate an intermediate flyby of Venus, with launch dates ranging from 2010 to 2025 . A notable trajectory from their study was found (with launch date in 2014 - clearly too soon for Inspiration Mars) that met all of the energy and TOF constraints within NASA's Design Reference Mission, ${ }^{6,7}$ and was proposed as a candidate for a near term human mission to Mars.

In 2010, Foster and Daniels ${ }^{12}$ investigated round-trip trajectories to Near-Earth objects (NEOs), Venus, and Mars (including trajectories to Mars with an intermediate Venus flyby). The trajectories found use impulsive maneuvers at each encounter (i.e. a powered flyby) and are therefore not free returns, but they do allow a spacecraft to return to Earth using maneuvers that are feasible with present day chemical propulsion systems.

In 2013, Bailey et al. ${ }^{14}$ and Folta et al. ${ }^{15}$ found fast round-trip trajectories to Mars with short stay-times. Their study includes flybys of Venus (both before and after the Mars encounter) and achieves short transfer times with the use of powered flybys and on-orbit staging. Many solutions were found with a total mission duration of about 1 year.

## II. Methodology

## A. Satellite Tour Design Program (STOUR)

The STOUR program (developed by the Jet Propulsion Laboratory ${ }^{16}$ and Purdue University ${ }^{17,18}$ ) was used to compute the Mars free-return opportunities with an intermediate Venus flyby. The STOUR program uses a patched-conic model with an analytic ephemeris to rapidly compute multiple gravity-assist trajectories. It imposes a grid search to find trajectories by stepping through specified launch dates and launch $V_{\infty}$ —thereby revealing all candidate trajectories within the search parameters.

From the list of candidate trajectories found using STOUR, the most desirable trajectories are selected based on parameters such as launch date, launch $V_{\infty}$, arrival $V_{\infty}$, and TOF. Of these "best case" opportunities, free-returns that provide characteristics comparable to those for the Inspiration Mars mission can be identified as potential candidates for a similar, IM-type, Mars flyby mission, or perhaps will provide a second chance opportunity for the Inspiration Mars mission itself.

The nominal Inspiration Mars trajectory (with launch date in 2018) was used as a baseline in determining the upper bounds for acceptable launch $V_{\infty}$, arrival $V_{\infty}$, and TOF values (as reported by Tito et al. ${ }^{13}$ ). Any trajectories found with launch $V_{\infty}$, arrival $V_{\infty}$, and TOF values within the upper bounds were considered as candidates, however some trajectories are selected as being more desirable than others. Table 1 shows the constraints used for the trajectory search in STOUR. The search parameters show launch dates over a 45 -year period, however it should be noted that (as stated by Okutsu et al. ${ }^{11}$ ) the inertial geometry of the planets Earth, Venus, and Mars approximately repeats every 32 years.

It should also be noted that the Space Launch System (SLS), which is expected to launch its first human

Table 1. Trajectory Search Parameters

| Parameter | Value |
| :--- | ---: |
| Max $V_{\infty, \text { Launch }}(\mathrm{km} / \mathrm{s})$ | 6.5 |
| Max $V_{\infty, \text { Arrival }}(\mathrm{km} / \mathrm{s})$ | 9.0 |
| Max TOF (days) | 600 |
| Min Launch Date (mm/dd/yyyy) | $12 / 01 / 2014$ |
| Max Launch Date (mm/dd/yyyy) | $01 / 31 / 2060$ |

crew in 2021, is estimated to be capable of launching a payload mass of over 20 metric tons with a $V_{\infty}$ of 6.5 $\mathrm{km} / \mathrm{s} .{ }^{19}$ Such a launch capability depends on the choice of upper stage, but more importantly is sufficient for the expected payload mass of the Inspiration Mars mission.

## B. Tisserand Graph

For the Mars free-return gravity-assist combinations (or paths) considered in this study [Earth-Venus-MarsEarth (EVME), Earth-Mars-Venus-Earth (EMVE), and Earth-Venus-Mars-Venus-Earth (EVMVE)] the feasibility of each path can first be investigated with the use of a Tisserand Graph. The Tisserand graph is a plot of orbital specific energy (or orbital period) versus radius of periapsis (assuming all planets have circular and coplanar orbits), and provides a graphical means of identifying (from an energy perspective) the feasibility of a gravity-assist path. A flyby of a gravitational body (e.g. Mars) is represented on the Tisserand graph, by plotting a curve for a chosen value of $V_{\infty}$ for all possible gravity-assist turn angles. Thus, for a set of bodies (e.g. Earth, Mars, and Venus), the Tisserand graph provides a plot of curves of constant $V_{\infty}$ for each body. A curve of constant $V_{\infty}$ is also of constant Tisserand parameter-hence the name Tisserand graph.

To illustrate the use of the Tisserand graph, an example is shown in figure 1 for the path EVME. Each curve in the plot represents constant $V_{\infty}$ for odd integer values (in units of $\mathrm{km} / \mathrm{s}$ ) from right to left on the plot (i.e. $1 \mathrm{~km} / \mathrm{s}, 3 \mathrm{~km} / \mathrm{s}, 5 \mathrm{~km} / \mathrm{s}$, etc.). The bold line traced out on the plot shows that for an Earth launch $V_{\infty}$ of $5 \mathrm{~km} / \mathrm{s}$, it is possible (energetically) to reach Venus, then flyby Mars, and finally return to Earth with an arrival $V_{\infty}$ of $5 \mathrm{~km} / \mathrm{s}$ (coincidentally the same as the $V_{\infty}$ at Earth launch). Note that the intersection of two curves means that a trajectory exists that connects the two planets with the indicated $V_{\infty}$ values at each encounter. Also note that following along a curve represents the energy change during the flyby. The dots shown on each curve represent the maximum amount of energy change possible for a minimum flyby altitude of 200 km . The derivation of the Tisserand graph and how it is used for investigating candidate gravity-assist paths is given by Strange and Longuski ${ }^{20}$ and Labunsky et al. ${ }^{21}$

## C. Pareto Set

For the case of near-term trajectories, no single trajectory has a minimum of all three parameters: TOF, $V_{\infty, \text { Launch }}$, and $V_{\infty, \text { Arrival }}$. Therefore, the Pareto optimal set (or Pareto set) of trajectories are identified as the "best" candidates for backup to the Inspiration Mars (or similar IM-type) mission.

For a given trajectory, the design "objectives" (for this study) are TOF, $V_{\infty, \text { Launch }}$, and $V_{\infty, \text { Arrival }}$. When comparing two trajectories (e.g. Trajectory A and Trajectory B) from the set of all near-term candidates, Trajectory A is said to dominate Trajectory B if all objectives in Trajectory A are less than or equal to (with at least one objective strictly less than) the objectives of Trajectory B. Otherwise, the two trajectories are nondominated. For example, trajectories A and B are nondominated (with respect to each other) if Trajectory A has a lower TOF than Trajectory B, but Trajectory B has a lower $V_{\infty, \text { Launch }}$ than Trajectory A.

By comparing all near-term trajectories found in STOUR, the remaining nondominated trajectories (i.e. the trajectories that are not dominated by any other trajectory in the near-term STOUR results) make up the Pareto optimal set-any of which could be argued as the best case. A complete discussion of Pareto optimal sets is given by Arora. ${ }^{22}$


Figure 1. This Tisserand graph example shows that a free-return trajectory is possible with the gravity-assist path EVME. To follow the path on the graph, Earth launch with $V_{\infty}$ of $5 \mathrm{~km} / \mathrm{s}$ occurs along the third (from the right) blue curve. The intersection of the bold black curve indicates that a trajectory exists from Earth to Venus with an arrival $V_{\infty}$ at Venus of $7 \mathrm{~km} / \mathrm{s}$. Following along the bold black curve represents the energy increase gained through the Venus gravity assist. The intersection of the bold black and red curves indicates the existence of a trajectory between Venus and Mars, with arrival $V_{\infty}$ at Mars of $5 \mathrm{~km} / \mathrm{s}$. The gravity assist at Mars (following the bold red curve) shows that a return trajectory to Earth is possible (since the red curve intersects Earth's blue curve) with an Earth arrival $V_{\infty}$ of $5 \mathrm{~km} / \mathrm{s}$.

## III. Results

Using the STOUR program, Mars free-return trajectories for the gravity-assist paths Earth-Venus-MarsEarth (EVME), Earth-Mars-Venus-Earth (EMVE), and Earth-Venus-Mars-Venus-Earth (EVMVE) were investigated. The results for the path EVME are shown in figure 2. The search included Earth launch $V_{\infty}$ from $2.5 \mathrm{~km} / \mathrm{s}$ to $6.5 \mathrm{~km} / \mathrm{s}$ (in steps of $0.25 \mathrm{~km} / \mathrm{s}$ ), with a minimum allowed altitude of 200 km at both Venus and Mars. Note that results are shown for TOF as large as 700 days for purposes of observing the broader design space, however, only opportunities with TOF of 600 days or less are considered for an IM-type mission.

Although the STOUR program is stepping through launch dates with 1-day increments (a relatively small step size for trajectory design), the results of figure 2 show trajectories clustered around specific launch dates. This clustering implies that, in order to meet the constraints imposed for this trajectory search, the design space is sensitive to launch date. The clusters of trajectories appear as seven distinct vertical stripes, occurring (from left to right in the figure) in 2017, 2021, 2034, 2036, 2047, 2049, and 2053. Of these opportunities, none in 2017 nor 2049 have any solutions with TOF below 600 days, and are therefore not candidates for an IM-type mission.

With several EVME candidate trajectories available, the results of figure 2 are investigated further with regard to Earth arrival $V_{\infty}$ (since this is a key parameter not explicitly shown in figure 2 ). Figure 3 shows Earth arrival $V_{\infty}$ on the horizontal axis, with TOF and Earth launch $V_{\infty}$ on the vertical axis and color bar, respectively. Because the figure no longer shows launch date, the launch year for notable trajectories (with emphasis on reduced Earth arrival $V_{\infty}$ ) is indicated in the figure for 2021, 2034, 2036, 2047, and 2053. The notable trajectories in 2021, 2034, and 2053 appear to be to have similar characteristics, and for the 2021 and 2053 trajectories, the 32-year time difference in launch date is consistent with the time for the inertial geometry of Earth, Mars, and Venus, to repeat (as discussed by Okutsu et al. ${ }^{11}$ ). Thus, the trajectories in 2053 are essentially a recurrence of the opportunities in 2021. (A detailed discussion on these reoccurring trajectories is given in section C.)

The opportunities in 2021 from figures 2 and 3 are the only realistic EVME candidates for a second chance to the Inspiration Mars mission since they are the only opportunities that occur in the near term


Figure 2. Mars free-return trajectories with intermediate Venus flyby before the Mars encounter (gravity assist path Earth-Venus-Mars-Earth). The time of flight from Earth launch to Earth arrival is shown on the vertical axis, the Earth launch date is shown on the horizontal axis, and the colorbar to the right of the plot indicates the launch $V_{\infty}$. All results shown have an Earth arrival $V_{\infty}$ less than or equal to $9 \mathrm{~km} / \mathrm{s}$.
(and after the nominal 2018 Inspiration Mars opportunity). Furthermore, the 2021 EVME trajectories are likely the only practical candidates for some other IM-type mission, as the purpose of such a mission is to pave the way for humans to explore Mars, and therefore is more significant if undergone in the near term.

It should also be noted that, as indicated in figure 3, the opportunities in 2036 and 2047 are near the maximum allowed Earth arrival $V_{\infty}$ of $9 \mathrm{~km} / \mathrm{s}$, and may therefore be less desirable when compared to other notable candidates in 2021, 2034, and 2053. The key characteristics of the trajectories identified in figure 3 are given in table 2. Note that $V_{\text {Entry }}$ refers to the inertial Earth entry speed at arrival, computed at 122 km altitude.

Table 2. Notable EVME Trajectories from Broad 45-Year Search

| Launch Date <br> $(\mathrm{mm} / \mathrm{dd} /$ yyyy $)$ | $V_{\infty, \text { Launch }}(\mathrm{km} / \mathrm{s})$ | C 3 <br> $\left(\mathrm{~km}^{2} / \mathrm{s}^{2}\right)$ | TOF <br> $($ days $)$ | $V_{\infty, \text { Arrival }}(\mathrm{km} / \mathrm{s})$ | $V_{\text {Entry }}$ <br> $(\mathrm{km} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $11 / 22 / 2021$ | 4.50 | 20.25 | 582 | 6.53 | 12.85 |
| $12 / 08 / 2021$ | 5.50 | 30.25 | 566 | 6.55 | 12.87 |
| $08 / 28 / 2034$ | 4.75 | 22.56 | 558 | 6.52 | 12.85 |
| $06 / 24 / 2036$ | 5.50 | 30.25 | 499 | 8.79 | 14.14 |
| $07 / 02 / 2047$ | 5.75 | 33.06 | 565 | 8.84 | 14.17 |
| $11 / 28 / 2053$ | 5.00 | 25.00 | 580 | 6.34 | 12.76 |

The free-return search results for the gravity-assist path Earth-Mars-Venus-Earth (EMVE) are very sparse in comparison to EVME. Because of the extremely low number of trajectories found, the search parameters were expanded slightly to accommodate a launch $V_{\infty}$ of up to $7.0 \mathrm{~km} / \mathrm{s}$. All other parameters in the search were kept the same as those used to obtain the EVME results. Despite the increase in allowable launch $V_{\infty}$ values, only 2 trajectories were found-both of which exhibited a TOF greater than 600 days. Thus, no trajectories from the EMVE search satisfied the constraints; thereby leaving EMVE an unlikely gravity-assist path for an IM-type mission.


Figure 3. Mars free-return trajectories with gravity assist path Earth-Venus-Mars-Earth. Trajectories shown in this plot are the same as those shown in figure 2, with Earth launch date exchanged for Earth arrival $V_{\infty}$ on the horizontal axis. Notable trajectories (with emphasis on low arrival $V_{\infty}$ ) are highlighted for each cluster of launch opportunities in 2021, 2034, 2036, 2047, and 2053.

For the gravity-assist path Earth-Venus-Mars-Venus-Earth (EVMVE), only near-term solutions were investigated, with launch dates ranging from $1 / 1 / 2018$ to $1 / 1 / 2030$. The constraints on launch and arrival $V_{\infty}$ imposed on the search were the same as those listed in table 1. The search results produced many solutions in 2021 and 2028, however, all of these solutions had TOF greater than 600 days, and therefore are not suitable candidates for an IM-type mission.

## A. Feasibility of Gravity-Assist Paths

One means to evaluate the feasibility of the path EVME versus EMVE is with the use of the Tisserand graph. As shown in the example use of the Tisserand graph in figure 1, the path EVME can provide trajectories with Earth launch $V_{\infty}$ of $5 \mathrm{~km} / \mathrm{s}$ and return to Earth with arrival $V_{\infty}$ as low as $5 \mathrm{~km} / \mathrm{s}$, which is not unlike the trajectories found in STOUR, as shown in figures 2 and 3 and in table 2. Therefore, it is clear from the Tisserand graph (and from the STOUR results) that path EVME is a promising candidate for providing solutions within the problem constraints.

The concerning issue is that the path EMVE can also be viewed in figure 1 by simply following the bold lines in reverse. Thus, the Tisserand graph shows that the path EMVE is also feasible with an Earth launch $V_{\infty}$ of $5 \mathrm{~km} / \mathrm{s}$ and Earth arrival $V_{\infty}$ of $5 \mathrm{~km} / \mathrm{s}$. The STOUR results however clearly show that EVME is the more feasible path with regards to satisfying the constraints set for this study. One key constraint however that the Tisserand graph does not show is TOF. To investigate this further, a new STOUR trajectory search was conducted for EMVE trajectories with TOF of up to 5 years (about 1826 days) -well beyond the set constraint of 600 days. The results in figure 4 show that by only extending TOF (and holding the launch and arrival $V_{\infty}$ constraints the same) many trajectories appear with comparable launch and arrival $V_{\infty}$ values to those found in the EVME search. All of the EMVE results however, have TOF longer than about 800 days-leaving no suitable EMVE candidates for an IM-type mission.

Since time is not represented on the Tisserand graph, Earth launch dates and stay times at Mars are also not visible from the Tisserand graph. The issue of Earth launch date not being shown can be disregarded since the STOUR simulations search over a span of launch dates beyond the 32 years estimated for the


Figure 4. Mars free-return trajectories with intermediate Venus flyby after the Mars encounter (gravity assist path Earth-Mars-Venus-Earth, or EMVE). TOF is shown for up to 5 years to show that opportunities do exist with similar energy requirements (at Earth launch and Earth arrival) compared to the EVME results (as indicated by the Tisserand graph). The EMVE opportunities shown in the figure however all exhibit TOF longer than about 800 days-unsuitable for an IM-type mission.
geometry of Earth, Mars, and Venus to repeat in inertial space. With regards to stay times at Mars, several studies that incorporate a stay time at Mars (such as Walberg, ${ }^{5}$ Casalino et al., ${ }^{8}$ Bailey et al., ${ }^{14}$ and Folta et al. ${ }^{15}$ ) have found solutions with the path EMVE, however such opportunities use a maneuver to capture into orbit at Mars, and are not suitable for an IM-type mission.

## B. Near-Term Opportunities

Of the opportunities found (shown in figures 2 and 3 ), all launch dates near or beyond 2034 are too distant for a mission like Inspiration Mars, since the primary purpose of such a mission is to send humans to Mars in the near future. Thus, the remaining near-term opportunities all have launch dates in 2021, which all occur within about a one-month time span (from November 18, 2021 to December 21, 2021).

Figures 5 and 6 show the near-term opportunities, with relatively small search step sizes (one day for launch date and $0.1 \mathrm{~km} / \mathrm{s}$ for launch $V_{\infty}$ ) to show more precisely the available opportunities in the near term. Since all launch dates in these figures occur around the same day, figure 6 shows arrival $V_{\infty}$ on the horizontal axis in place of launch date.

In figures 5 and 6 , no single opportunity exists in the data set that minimizes TOF, Earth launch $V_{\infty}$, and Earth arrival $V_{\infty}$ simultaneously. Therefore, the Pareto set (as discussed in Sec. C) is found to characterize the set of "best" cases. The trajectories that make up the Pareto set are circled in figure 6, and since none are dominated by any other trajectories shown in the near-term results, are all arguably the "best" near-term opportunities. The near-term Pareto-set trajectories are listed in table 3. Note that for the circled opportunity with an index of 20 (found near the lower left corner of figure 6), the TOF (when rounded to the nearest day), was found to be equal to the TOF of the 12/17/2021 opportunity. Thus, by treating these two TOF as equal, the circled trajectory with index of 20 is dominated by the $12 / 17 / 2021$ opportunity in both launch $V_{\infty}$ and arrival $V_{\infty}$, and is therefore not listed in table 3 .

Of the best opportunities listed in table 3 , some stand out as potentially more desirable. For example, the opportunity on $11 / 22 / 2021$ has the lowest launch $V_{\infty}$ and nearly the lowest arrival $V_{\infty}$, but has the longest TOF. Conversely, the opportunity on $12 / 19 / 2021$ has the shortest TOF, but has the largest launch $V_{\infty}$ and nearly the largest arrival $V_{\infty}$. The opportunity with the smallest arrival $V_{\infty}$ (occurring on $12 / 04 / 2021$ ) has


Figure 5. Near-term EVME Mars free-return trajectories with launch dates spanning from November 18 to December 21 of 2021. The time of flight from Earth launch to Earth arrival is shown on the vertical axis, the Earth launch date is shown on the horizontal axis, and the colorbar to the right of the plot indicates the launch $V_{\infty}$. All results shown have an Earth arrival $V_{\infty}$ less than or equal to $9 \mathrm{~km} / \mathrm{s}$.


Figure 6. Near-term EVME Mars free-return trajectories with launch dates spanning from November 18 to December 21 of 2021. Trajectories shown in this plot are the same as those shown in figure 5, with Earth launch date exchanged for Earth arrival $V_{\infty}$ on the horizontal axis. Circled trajectories are considered to be the best near-term candidates (as defined by a Pareto optimal set), and are listed explicitly in table 3.

Table 3. Best Near-Term Opportunities

| Launch Date <br> $(\mathrm{mm} / \mathrm{dd} /$ yyyy $)$ | $V_{\infty, \text { Launch }}$ <br> $(\mathrm{km} / \mathrm{s})$ | C 3 <br> $\left(\mathrm{~km}^{2} / \mathrm{s}^{2}\right)$ | TOF <br> $($ days $)$ | $V_{\infty, \text { Arrival }}$ <br> $(\mathrm{km} / \mathrm{s})$ | $V_{\text {Entry }}$ <br> $(\mathrm{km} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $11 / 22 / 2021$ | 4.50 | 20.25 | 582 | 6.53 | 12.85 |
| $12 / 03 / 2021$ | 5.10 | 26.01 | 579 | 6.95 | 13.07 |
| $12 / 04 / 2021$ | 5.20 | 27.04 | 569 | 6.46 | 12.82 |
| $12 / 08 / 2021$ | 5.50 | 30.25 | 566 | 6.55 | 12.87 |
| $12 / 14 / 2021$ | 6.00 | 36.00 | 564 | 6.81 | 13.00 |
| $12 / 15 / 2021$ | 6.10 | 37.21 | 563 | 6.78 | 12.99 |
| $12 / 16 / 2021$ | 6.20 | 38.44 | 561 | 6.77 | 12.98 |
| $12 / 17 / 2021$ | 6.30 | 39.69 | 560 | 6.78 | 12.99 |
| $12 / 19 / 2021$ | 6.50 | 42.25 | 559 | 6.84 | 13.02 |

launch $V_{\infty}$ and TOF values that lie near the middle of the spread of launch $V_{\infty}$ and TOF values listed in the Pareto set. The truly most desirable near-term opportunity will ultimately depend on how the importance of minimizing each parameter is weighted in the final mission design.

The trajectory with launch date on $11 / 22 / 2021$ is shown in figure 7 , and due to the relatively similar launch and encounter dates, is representative of all trajectories in the near-term Pareto set. All trajectories in the Pareto set exist primarily in the ecliptic plane, where the only arc with notable inclination (in each trajectory) occurs on the Earth-Venus legs with a value of approximately 5.4 ${ }^{\circ}$.


Figure 7. EVME opportunity generated by STOUR, with launch date on $11 / 22 / 2021$ (from the near-term trajectory Pareto set). The launch, encounter, and arrival dates are annotated on the figure. Although the majority of the trajectory is approximately in the ecliptic plane, the Earth-Venus leg has an inclination of $5.4^{\circ}$ (into the page in the view above).

To demonstrate that the EVME trajectories generated by STOUR (using patched conics with an analytic ephemeris) are valid in a high-fidelity model (such as considering the gravity force due to multiple bodies simultaneously), the STOUR trajectory with launch date on $11 / 22 / 2021$ was reproduced using AGI's Systems Tool Kit (STK) with the propagator Astrogator. The high-fidelity STK propagation accounts for the gravity of the Earth, Moon, Sun, Mars, and Venus throughout the trajectory (with the exeption that the gravity of Mars and Venus are neglected near Earth launch and Earth arrival).The resulting STK trajectory is shown
in figure 8 and clearly resembles the STOUR trajectory in figure 7.


Figure 8. STK plot of EVME opportunity (propagated using STK's Astrogator), with launch date on $11 / 22 / 2021$. The launch, encounter, and arrival dates are annotated on the figure. By comparison with the STOUR-generated trajectory in figure 7, the STK trajectory is clearly similar. The annotations show that the launch, encounter, and arrival dates are the same as those found in the STOUR trajectory with the exception of the one-day difference in Mars encounter date.

Parameters from the STK trajectory are shown in table 4 along with those from the STOUR solution for comparison. The similarity of the STK and STOUR solution values indicates that (for the gravity-assist trajectories found in this study) the patched-conic method with analytic ephemeris (as used in STOUR) provides solutions that closely approximate the true dynamic motion. The similarity of the results in table 4 also validates the use of STOUR-generated trajectories as initial guesses for targeting high-fidelity solutions in tools such as STK.

Table 4. High-Fidelity Comparison of 11/22/2021 EVME Opportunity

| Propagator | Launch Date <br> $(\mathrm{mm} / \mathrm{dd} /$ yyyy $)$ | $V_{\infty, \text { Launch }}(\mathrm{km} / \mathrm{s})$ | C 3 <br> $\left(\mathrm{~km}^{2} / \mathrm{s}^{2}\right)$ | $h_{V}$ <br> $(\mathrm{~km})$ | $h_{M}$ <br> $(\mathrm{~km})$ | TOF <br> $($ days $)$ | $V_{\infty, \text { Arrival }}(\mathrm{km} / \mathrm{s})$ | $V_{\text {Entry }}$ <br> $(\mathrm{km} / \mathrm{s})$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Astrogator | $11 / 22 / 2021$ | 4.511 | 20.353 | 11097 | 363 | 582.5 | 6.459 | 12.87 |
| STOUR | $11 / 22 / 2021$ | 4.500 | 20.250 | 10868 | 346 | 582.2 | 6.526 | 12.85 |

## C. Physical Behavior and Uniqueness of Candidate Trajectories

For each leg (i.e. each trajectory arc between bodies) of the near-term Pareto set trajectories in table 3, the transfer angle is nearly $180^{\circ}$-a characteristic resemblant of a Hohmann transfer—and is visible in figure 7 (with a slightly larger than $180^{\circ}$ transfer shown on the Mars-Earth leg). Such Hohmann-like behavior may be the characteristic which explains why the opportunities found have relatively low $V_{\infty}$ at Earth launch and arrival, however, it may also be a contributing factor to the relatively short span of available launch dates for near-term opportunities.

Since the orbits of Venus and Mars are slightly inclined with respect to the ecliptic (and are therefore not in the same plane as Earth's orbit about the Sun), a transfer angle of $180^{\circ}$ between any of these bodies will (in general) result in a transfer arc that is inclined by nearly $90^{\circ}$ with respect to the ecliptic. This out-of-plane effect on the transfer arc is due to the position vectors for the departure and target bodies (which are used to compute the connecting Lambert arc) lying in a plane that is approximately normal to the ecliptic-a direct consequence of the relative inclination of the planetary orbits. Such an out-of-plane transfer requires large $V_{\infty}$ at Earth launch (or arrival), and (if a plane change back to the ecliptic is required after a flyby) will require a gravity-assist turning angle too large for Venus or Mars to provide. Of course,
in the planar case, a $180^{\circ}$ transfer is desirable since that is the condition for the (impulsive) $\Delta V$-optimal Hohmann transfer.

For the near-term EVME results listed in table 3, (as well as for the candidate opportunities in 2034 and 2053) it appears that transfer angels near $180^{\circ}$ are needed to provide the low Earth launch and arrival $V_{\infty}$ values. (Note that this also gives reason as to why the low launch and arrival $V_{\infty}$ candidates all have similar TOF-around 570 days.) Such a requirement on transfer angle will result in a sensitivity to launch, encounter, and arrival dates, since getting too close to (or deviating too far from) the $180^{\circ}$ transfer angle geometry, is expected to increase launch and arrival $V_{\infty}$, or perhaps, not exist at all due to insufficient bending capability at either Venus or Mars (for large plane changes between arcs).

As mentioned previously, the 32-year time difference between the 2021 and 2053 opportunities agrees with the estimate of Okutsu et al. ${ }^{11}$ as the approximate time for the orientations of Earth, Venus, and Mars to repeat in inertial space. Thus, it is expected that the same physical factors are in play for both the 2021 and 2053 opportunities, and by the same logic, that opportunities similar to those in 2034 will reoccur 32 years later, in 2066. This repetition cycle suggests that the opportunities found in 2021, 2034, and 2053, are dependent on some inertially fixed characteristics (such as eccentricity, inclination, and ascending and descending node locations) of the Earth, Venus, and Mars orbits, and that these inertial characteristics are also playing a significant role in reducing the $V_{\infty}$ at Earth launch and arrival.

One means to observe what role (if any) eccentricity plays on a gravity-assist trajectory, is in a radial distance plot. Figures 9 and 10 show radial distance plots for the $11 / 22 / 2021$ and $8 / 28 / 2034$ opportunities, respectively. The figures show the radial distance (with respect to the Sun) of each planet (Earth, Venus, and Mars), as well as for the spacecraft's trajectory, plotted against time. Each leg of the spacecraft trajectory is shown in alternating colors to indicate where in the arc the gravity assist occurs. To investigate the node crossings of Venus and Mars, the ascending and descending nodes for each planet's orbit are indicated by the dots shown along the curves for Venus and Mars. Additionally, the inclination of each leg of the spacecraft's trajectory is annotated next to the corresponding curve on the plot.


Figure 9. Radial distance plot of EVME opportunity with launch date on $11 / 22 / 2021$. The plot shows the eccentric orbits of Venus, Earth, and Mars along with the spacecraft trajectory. The dots on the Venus and Mars curves indicate a node crossing, and a color change on the spacecraft trajectory curve indicates a transition onto a new transfer arc (which occurs at the Venus and Mars encounters). The plot shows departure near Earth's perihelion and arrival near Earth's aphelion. The Venus and Mars encounters also occur near an apse of the spacecrafts transfer arcs, as well as near a node crossing.

When looking at each radial distance plot, it is important to note two key characteristics that are known to reduce the energy required for a transfer between two bodies. The first involves a concept discussed by Lawden ${ }^{23}$ for transferring between two elliptical orbits (assuming collinear lines of apsides). In his discussion, Lawden shows that for an (impulsive) $\Delta V$-optimal transfer from one ellipse to another, the spacecraft must enter (and depart) the transfer arc tangentially, and at an apse (of the departure ellipse, the transfer ellipse,


Figure 10. Radial distance plot of EVME opportunity with launch date on $8 / 28 / 2034$. The plot shows that at the Venus encounter, the arrival and departure transfer arcs are near perihelion, as well as Venus itself. At the Mars encounter, the arrival transfer arc is near aphelion, and Mars is at perihelion. Additionally, both the Venus and Mars encounters occur near their node crossings.
and arrival ellipse).
The second characteristic relates back to the issue of large transfer arc inclinations for transfer angles near $180^{\circ}$. Recall that this issue only occurs when the departure and arrival bodies' orbits are in different planes. Such a problem does not exist however if encounters occur at points where these orbital planes intersect. For the case of Earth, Venus, and Mars, this occurs where Venus and Mars cross the ecliptic-at their ascending and descending nodes. Therefore, the desired characteristics to observe in figures 9 and 10, are that transfer arcs start and end at an apse (either of the planet's orbit, the spacecraft's orbit, or ideally at both), and that the Venus and Mars encounters occur at a node crossing to allow for transfer arcs near $180^{\circ}$.

Figure 9 shows (from left to right on the plot) that the spacecraft departs Earth near Earth's perihelion, encounters Venus near the spacecraft's perihelion, encounters Mars near the spacecraft's aphelia (both on the arrival and departure transfer arcs), and finally encounters Earth at Earth's aphelion. Additionally, the figure shows that the Venus and Mars encounters both occur near their respective node crossings, which explains why the transfer arcs with nearly $180^{\circ}$ transfer angles can exist with relatively small inclination. We also see from the figure that the Venus and Mars encounters do not occur very near to either the perihelion or aphelion of Venus' or Mars' orbits, nor do we depart Earth or arrive at Earth near either apse of the spacecraft's transfer arcs. Nevertheless, several transfers do begin and end near an apse, and both encounters occur near Venus' and Mars' node crossings.

Figure 10 shows no apse at Earth departure (neither from Earth's nor the spacecraft's orbits), but at the Venus encounter, all three orbits (the spacecraft's arrival and departure arcs, and that of Venus) are at their respective perihelia. At the following Mars encounter, Mars is at perihelion and the spacecrafts arrival arc is near aphelion. The final arrival at Earth however does not occur near an apse. The figure also shows that the Venus and Mars encounters occur very near the node crossings, which explains (as with the 2021 case) why such low transfer arc inclinations can exist with near $180^{\circ}$ transfer angles. To better illustrate that the 2034 trajectory has near $180^{\circ}$ transfer arcs (with a slightly larger transfer angle for the Mars-Earth leg), its trajectory is shown in figure 11.

It should be noted that the issue of $180^{\circ}$ transfer angels producing highly inclined trajectories can be resolved with the use of a broken-plane maneuver. Such a maneuver is typically small (particularly if applied to this problem since the inclinations of Venus' and Mars' orbits are small), however, the use of a brokenplane maneuver prevents the resulting EVME trajectory from being a true free return, and is therefore less desirable for an IM-type mission.


Figure 11. Trajectory plot of an EVME opportunity with launch date on $8 / 28 / 2034$. The plot shows that all transfer arcs have transfer angles that are near $180^{\circ}$ (although slightly larger on the Mars-Earth leg). Such a trajectory is possible (ballistically) with small inclination transfers due to the Venus and Mars encounters occurring near a node crossing. This same effect is also in play for the opportunities in 2021, of which a representative case is shown in figure 7 .

Nevertheless, a broken-plane maneuver could potentially allow for transfers closer to a true Hohmann transfer, and thereby (potentially) provide lower Earth launch and arrival $V_{\infty}$ values than those found in table 3. Such trajectories are not investigated in this paper, as the primary goal of this study is to find trajectories for an IM-type mission that are purely ballistic-allowing those aboard the spacecraft to return to Earth without the need of deterministic maneuvers. Therefore, it is not expected that a broken-plane maneuver be used for an IM-type mission, and the purpose of investigating such trajectories would be purely to obtain more insight into the behavior of the available design space.

## D. The Case for Venus

Free-return trajectories to Venus have also been investigated. In 1969 Hollister, ${ }^{24}$ and in 1970 Hollister and Menning, ${ }^{25}$ found free-return trajectories that repeatedly encounter Earth and Venus (also known as Earth-Venus cycler trajectories). In 2000, Bonfiglio et al. ${ }^{26}$ investigated Venus free returns using aerogravity assists. In general however, far less attention has been paid on human missions to Venus as it has been for Mars. Nevertheless, the case for Venus (a play on the book title The Case for Mars by Zubrin and Wagner ${ }^{27}$ ) has been proposed (or at least addressed) by some. ${ }^{28-32}$

For Inspiration Mars, it is clear from this study that (apart from the nominal Inspiration Mars trajectory in early 2018) the only viable IM-type mission candidates that exist in the near term are the EVME trajectories in 2021. However, as the 2018 nominal launch date nears, it may turn out that the 2021 EVME opportunity becomes the new "nominal" case, and if so, there would be no near-term backup for such a mission. Since a primary goal of Inspiration Mars is to push human spaceflight into deep space, many of the mission objectives could still be met with a human flyby of Venus alone - albeit an Inspiration Venus mission.

To investigate such an option, the Venus free-return gravity-assist path Earth-Venus-Earth (EVE) was searched using STOUR, and many near-term opportunities were found. Because this is proposed as an alternate for Inspiration Mars (or some other IM-type mission), the same constraints were imposed on this STOUR search with regard to Earth launch and arrival $V_{\infty}$ and TOF. Therefore, any vehicle designed for the EVME 2021 trajectory, should be capable of flying a candidate EVE trajectory.

Figure 12 shows the search results over launch dates spanning from 1/1/2019 through $1 / 1 / 2027$ with steps of one day, and all with Earth arrival $V_{\infty}$ at or below $9 \mathrm{~km} / \mathrm{s}$. These dates were chosen as the most
relevant near-term launch dates for backup to the nominal Inspiration Mars trajectory, as well as to the 2021 EVME opportunities. An eight-year span of launch dates was chosen as it is the approximate time for the Earth-Venus geometry to repeat in inertial space (approximately 8 Earth and 13 Venus revolutions about the Sun). The figure shows five columns of opportunities whose launch dates each span about six months. The distance between columns is consistent with the Earth-Venus synodic period (assuming circular-coplanar orbits) of about 1.6 years, which repeats approximately five times in the eight years shown. From the results in the figure, it is clear that many EVE candidates are available for an IM-type mission.


Figure 12. Venus free-return trajectories (gravity assist path Earth-Venus-Earth) with launch dates spanning eight years, from January 1, 2019 to January 1, 2027. The time of flight from Earth launch to Earth arrival is shown on the vertical axis, the Earth launch date is shown on the horizontal axis, and the colorbar to the right of the plot indicates the launch $V_{\infty}$. All results shown have an Earth arrival $V_{\infty}$ less than or equal to 9 km/s.

To identify the most desirable EVE near-term candidates, figure 13 shows the same candidate trajectories as shown in figure 12, but with Earth launch date exchanged for Earth arrival $V_{\infty}$ on the horizontal axis. The trajectories that make up the Pareto set have been circled, and are mostly made up of opportunities in 2026, although those that exhibit lower arrival $V_{\infty}$ have launch dates in 2019 and 2021 (as annotated on the figure).

A representative subset of the Pareto-set EVE trajectories is given in table 5. The cases listed include a case with the overall lowest Earth launch $V_{\infty}$, Earth arrival $V_{\infty}$, and TOF, respectively. Since the Pareto set does not include many of the candidates from other launch years, the case from 2019 is included in the list. Additionally, candidates that more closely follow the 2021 EVME launch dates (such as in 2023) are not included in the Pareto set, but due to more convenient launch dates, may be of interest for backup to the EVME candidates in 2021. Therefore, one such EVE case with launch date in 2023 is also included in table 5.

One immediate observation of the results in table 5 is that the EVE trajectories are capable of providing much lower launch and arrival $V_{\infty}$ and TOF values than the EVME cases. The difference is even more extreme when compared to the nominal 2018 Inspiration Mars trajectory. In fact, the EVE trajectories appear so attractive that a Venus only flyby mission may be more desirable than the EVME opprtunity as a backup, or in the extreme case, a replacement for the nominal Inspiration Mars trajectory in early 2018.

With 2018 launch dates in mind, recall that the inertial repeat time for the EVE opportunities approximately occurs every eight years. Since the Pareto set opportunities of figure 13 were mostly populated by


Figure 13. Venus free-return trajectories (the same as those shown in figure 2) with Earth launch date exchanged for Earth arrival $V_{\infty}$ on the horizontal axis. Candidates in the Pareto set are indicated with a circle, and their respective launch year is indicated by the annotations shown. Most Pareto-set candidates are shown to occur in the year 2026.

Table 5. Subset of Best Near-Term EVE Opportunities

| Launch Date <br> $(\mathrm{mm} / \mathrm{dd} /$ yyyy $)$ | $V_{\infty, \text { Launch }}$ <br> $(\mathrm{km} / \mathrm{s})$ | C 3 <br> $\left(\mathrm{~km}^{2} / \mathrm{s}^{2}\right)$ | TOF <br> $($ days $)$ | $V_{\infty, \text { Arrival }}$ <br> $(\mathrm{km} / \mathrm{s})$ | $V_{\text {Entry }}$ <br> $(\mathrm{km} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $07 / 31 / 2026$ | 2.75 | 7.56 | 366 | 7.09 | 13.15 |
| $05 / 07 / 2021$ | 6.50 | 42.25 | 400 | 4.34 | 11.90 |
| $09 / 18 / 2026$ | 6.25 | 39.06 | 327 | 7.25 | 13.24 |
| $11 / 18 / 2019$ | 6.00 | 36.00 | 417 | 6.39 | 12.79 |
| ${ }^{*} 07 / 02 / 2023$ | 5.75 | 33.06 | 347 | 7.83 | 13.56 |

*Not in Pareto optimal set of trajectories from figure 13.

2026 launch dates, this implies that in 2018 (eight years prior to 2026) similarly attractive opportunities should be expected. To investigate this further, the STOUR results for EVE candidate trajectories in 2018 are shown in figure 14. The search was over launch dates from $1 / 1 / 2018$ to $1 / 1 / 2019$, and trajectories that make up the Pareto set of 2018's best EVE cases are indicated with a circle. Representative cases of the 2018 EVE Pareto set are listed in table 6. The cases given represent the lowest Earth launch $V_{\infty}$, arrival $V_{\infty}$, and TOF, respectively. Note that the cases on $8 / 3 / 2018$ and $9 / 17 / 2018$ appear to have approximately the same values as those on $7 / 31 / 2026$ and $9 / 18 / 2026$ (in table 5 ), respectively, and are likely repetitions of the same opportunities.


Figure 14. Venus free-return trajectories in 2018 (with launch dates spanning from January 1,2018 to January 1, 2019). Candidates in the 2018 Pareto set are indicated with a circle, and appear to have similar characteristics to those observed in 2026 (as is expected due to the eight-year cycle for Earth-Venus geometry to approximately repeat in inertial space).

Table 6. Subset of Best 2018 EVE Opportunities

| Launch Date <br> $(\mathrm{mm} / \mathrm{dd} /$ yyyy $)$ | $V_{\infty, \text { Launch }}$ <br> $(\mathrm{km} / \mathrm{s})$ | C 3 <br> $\left(\mathrm{~km}^{2} / \mathrm{s}^{2}\right)$ | TOF <br> $($ days $)$ | $V_{\infty, \text { Arrival }}$ <br> $(\mathrm{km} / \mathrm{s})$ | $V_{\text {Entry }}$ <br> $(\mathrm{km} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $08 / 03 / 2018$ | 2.75 | 7.56 | 371 | 7.13 | 13.17 |
| $02 / 23 / 2018$ | 6.50 | 42.25 | 454 | 6.35 | 12.77 |
| $09 / 17 / 2018$ | 5.75 | 33.06 | 324 | 7.18 | 13.20 |

## IV. Conclusion

This study has found key opportunities, such as the EVME Mars free-return on November 22, 2021, for application to Inspiration Mars (or similar IM-type) mission. These trajectories were found to possess significantly lower $V_{\infty}$ values at Earth launch and arrival (with a moderate increase in TOF) compared to those exhibited by the nominal Inspiration Mars trajectory (with launch date in late 2017 - early 2018).

All trajectories of the gravity-assist path EMVE were found to provide similar launch and arrival $V_{\infty}$
values, but all with TOF greater than about 800 days-much too long for an IM-type mission. Similarly, the path EVMVE produced solutions with acceptable launch and arrival $V_{\infty}$, but all with TOF longer than the 600-day constraint.

The existence of the low launch and arrival $V_{\infty}$ values found from the EVME search results can be attributed to the Hohmann-like transfer arcs between bodies. The Earth-Venus-Mars geometry that allows for such opportunities occurs in 2021 and 2034, whereupon each set of solutions approximately reoccurs every 32 years thereafter.

An EVME opportunity could also serve as a replacement for the nominal Inspiration Mars trajectory in 2018. In fact, the November 22, 2021 opportunity was recently proposed to the US House of Representatives Committee on Science, Space, and Technology, on February 27, 2014. The proposal was for an IM-type mission (using the November 2021 opportunity) to be the first human-crew, deep-space mission for the Orion and Space Launch System.

Venus free-return opportunities (using the gravity-assist path Earth-Venus-Earth) were also found with the same search constraints on Earth launch and arrival $V_{\infty}$, and TOF as those for the Mars free-return cases. Many candidate EVE opportunities were found with either lower launch $V_{\infty}$, arrival $V_{\infty}$, or TOF than any found for the EVME Mars flyby opportunities. Although Mars is the ultimate goal, the EVE opportunities may prove to be essential for an IM-type mission, with Venus as the primary flyby target, if a mission to flyby Mars becomes unobtainable by the desired launch dates. Although forgoing Mars is not ideal, a Venus free-return would allow for a human deep-space mission, and thereby meet many of the primary mission objectives of Inspiration Mars.

With regards to the Inspiration Mars mission itself, there is need for a second-chance opportunity to the nominal trajectory in 2018. Several near-term EVME opportunities found in this study are suitable candidates to (and with regards to launch and arrival $V_{\infty}$, more desirable than) the nominal trajectory. If the launch date in early 2018 cannot be met, the EVME gravity-assist path provides this opportunity in 2021-essential for keeping the Inspiration Mars mission alive.

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