# Preliminary Analysis of Establishing Cycler Trajectories Between Earth and Mars via $\mathrm{V}_{\infty}$ Leveraging 

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#### Abstract

Many cycler concepts have been proposed to provide safe and comfortable quarters for astronauts traveling between the Earth and Mars. However, no literature has appeared to show how these massive vehicles might be placed into their cycler trajectories. In this paper, we explore the use of $V_{\infty}$ leveraging to establish cycler vehicles in their desired orbits. In all cases, $V_{\infty}$ leveraging reduces the total $\Delta V$ to achieve the cycler orbit. In the case of the classic Aldrin cycler, the propellant savings can be as large as 23 metric tons for a cycler vehicle with a dry mass of 70 metric tons. The two-synodic period cyclers enjoy lesser gains from $V_{\infty}$ leveraging, but have a smaller total mass due to their low approach velocities at Earth and Mars. These characteristics make the two-synodic period cyclers attractive for an Earth-Mars human transportation system. If the one-way concept is selected, in which humans remain permanently on Mars, then the number of required two-synodic period cycler vehicles drops from four to two. The cycler concept may provide a crucial enabling technology that is safe, economical, and sustainable for the continuous habitation of Mars.


## Nomenclature

a $\quad=$ Semi-major axis, AU
e $\quad=$ Eccentricity
$\mathrm{h} \quad=$ Altitude, km
$\mathrm{K} \quad=$ Number of Earth orbit revolutions
L
$=$ Number of spacecraft orbit revolutions
$\mathrm{M} \quad=$ Spacecraft orbit revolution on which the maneuver performed
$\mathrm{V}_{\infty} \quad=$ Hyperbolic excess velocity, $\mathrm{km} / \mathrm{s}$
$\Delta \mathrm{V} \quad=$ Change in velocity, $\mathrm{km} / \mathrm{s}$
$\Delta \mathrm{V}_{D S M}=$ Deep space maneuver, $\mathrm{km} / \mathrm{s}$
$\Delta \mathrm{V}_{\text {total }}=$ Total $\Delta \mathrm{V}$ for $\mathrm{V}_{\infty}$-leveraging technique, $\mathrm{km} / \mathrm{s}$
$\Delta \mathrm{V}_{\text {direct }}=\Delta \mathrm{V}$ required to launch directly from low Earth orbit into cycler trajectory, $\mathrm{km} / \mathrm{s}$
superscripts
,$+-\quad=$ Earth encounter performed after $(+)$, before $(-)$ the spacecraft passes the line of apsides

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## I. Introduction

In the late 1960s and early 1970s, Hollister ${ }^{1,2}$ and Hollister and Menning ${ }^{3,4}$ discovered trajectories that use gravity-assists to repeatedly encounter the Earth and Venus. Rall ${ }^{5}$ and Rall and Hollister ${ }^{6}$ were the first to show that these types of trajectories also exist between Earth and Mars.

Trajectories that repeatedly encounter the same planets on a regular schedule without stopping are now known as cycler trajectories, or cyclers. There are ballistic cyclers, which use only a small amount of propellant, and powered cyclers, which require large propellant expenditures. There are also semi-cyclers, which involve occasional stops, that can be combined with cyclers to support continuous Mars habitation. ${ }^{7}$

The advantage of a ballistic cycler is that the vehicle can be more massive than a powered cycler. Massive vehicles are preferable for long human missions because they allow for increased space, and therefore comfort, for the crew. Cycler vehicles can also be made safer by adding a significant amount material between the source and the crew for radiation protection.

Perhaps the simplest and most famous of the Mars cyclers is the Aldrin cycler, named for Buzz Aldrin. ${ }^{8,9}$ Several other types of cycler trajectories have been offered as possible solutions by Niehoff; ${ }^{10,11}$ Friedlander et al. ${ }^{12}$ Brynes et al.; ${ }^{13}$ McConaghy et al.; ${ }^{14}$ and Russell and Ocampo. ${ }^{15}$ Each of these have their advantages and disadvantages, but the problem of getting cycler vehicles into their cycler orbits is significant, and establishing these transfer trajectories has not been investigated. In order to effectively compare each of the cycling trajectories, the initial mass in low-Earth orbit (IMLEO) cost of establishing these cycler trajectories must be taken into account.

## II. Methodology

Assuming a circular co-planar model, the simplest way to get onto the desired trajectory would be to perform a single, large maneuver at Earth, but this method would be expensive and non-optimal. In light of this, a more efficacious method of constructing "establishment trajectories" is attempted.

At this point, it is necessary to introduce new terminology for the trajectories involved. The new term "establishment-cycler" will henceforth be used to designate the two combined trajectories, namely the establishment trajectory and the cycler trajectory.

## A. $\mathrm{V}_{\infty}$ Leveraging

$\mathrm{V}_{\infty}$ leveraging is the use of a small deep-space maneuver to modify the $\mathrm{V}_{\infty}$ at a body. This maneuver, when used in conjunction with a gravity assist maneuver to get onto the desired cycler orbit, reduces the overall $\Delta \mathrm{V}$ for the mission. Hollenbeck ${ }^{16}$ was the first to use this technique when constructing a $\Delta \mathrm{V}$-EGA trajectory. The term $\mathrm{V}_{\infty}$-leveraging was first used by Williams and Longuski ${ }^{17}$ when describing $\Delta \mathrm{V}$-VGA trajectories and then more generally by Sims and Longuski. ${ }^{18}$

A detailed description of the exterior $\mathrm{V}_{\infty}$ leveraging is given by Sims et al. ${ }^{19}$ as follows and is illustrated in Fig. 1. First, a nominal orbit is chosen that has a period that will intercept the Earth after an integer number of periods. To find such nominal orbits, one chooses the number of Earth orbit revolutions (K) and the number of spacecraft orbit revolutions (L). If the spacecraft is on this orbit and no other maneuver is performed, the spacecraft would intercept the Earth tangentially in K years.

Next, an initial tangential $\Delta \mathrm{V}$ is performed so that the resulting $\mathrm{V}_{\infty}$ is slightly larger than the $\mathrm{V}_{\infty}$ required to get onto the nominal orbit.

On some revolution number (M) of the orbit, a retrograde maneuver is performed at aphelion. This maneuver causes the orbit of the spacecraft to intersect the orbit of the Earth non-tangentially twice, meaning that a gravity-assist maneuver can be used to raise (or lower) the energy of the orbit. The notation used to specify a given $\mathrm{V}_{\infty}$-leveraging technique is $\mathrm{K}: \mathrm{L}(\mathrm{M})^{ \pm}$.

Of course, just because the two orbits intersect does not mean that the Earth will be there at the same time as the spacecraft. Consequently, the retrograde $\Delta \mathrm{V}$ applied at aphelion must be iterated upon until the spacecraft and the Earth intersect.

The trajectory after the flyby is determined by whether the encounter is selected to be before (-) or $\operatorname{after}(+)$ perihelion and by the altitude of the flyby. Sims found ${ }^{19}$ that for aphelion radii at the orbital distance of Mars, flybys before perihelion require less total $\Delta \mathrm{V}$. The trajectories considered in this paper will all have aphelia close to Mars.


Figure 1. The $V_{\infty}$-leveraging maneuver consists of a (1) $\Delta V_{\text {launch }}$ followed by a (2) $\Delta V_{\text {aphelion }}$ to increases the $V_{\infty}$ at the subsequent Earth flyby, which may be chosen to occur before ( $3^{-}$) or after ( $3^{+}$) perihelion. It should be noted that the crew does not board the cycler vehicle until the Earth flyby.

## B. Genetic Algorithm

Finding an appropriate resulting trajectory using $\mathrm{V}_{\infty}$ leveraging can be time consuming, especially if we use a trial and error approach. One must initially choose the $\mathrm{K}, \mathrm{L}, \mathrm{M}$, the magnitude of the $\mathrm{V}_{\infty}$ above the nominal orbit, and the altitude of the flyby to determine the final characteristics and to see if they meet the requirements.

A genetic algorithm is implemented to minimize the total $\Delta \mathrm{V}$ subject to the constraints

$$
\begin{align*}
\left|a_{\text {desired }}-a_{\text {genetic }}\right| & \leq a_{\text {error }}  \tag{1}\\
\left|e_{\text {desired }}-e_{\text {genetic }}\right| & \leq e_{\text {error }}
\end{align*}
$$

where $\mathrm{a}_{\text {desired }}$ is the desired final semi-major axis, $\mathrm{a}_{\text {genetic }}$ is the actual value of the semi-major axis of the cycler orbit from the genetic algorithm, and $\mathrm{a}_{\text {error }}$ is the allowable difference between these two values. The eccentricity variables are similar.

MATLAB's ${ }^{\circledR}$ built-in genetic algorithm function, ga, is used with a design point population of 40 . There are three genetic algorithm operators that mimic the evolutionary process: selection, crossover, and mutation. The selection function (how the genetic algorithm chooses parents for the next iteration) is MATLAB's ${ }^{\circledR}$ stochastic uniform function. The crossover function (how two parents are combined to form a child) is MATLAB's ${ }^{\circledR}$ heuristic function. The mutation function (how small, random changes are made to children) is MATLAB's ${ }^{\circledR}$ adaptive feasible function.

The range of values used for $\mathrm{K}, \mathrm{L}$, and M were varied throughout the process. At first they were allowed to be as much as 15 to see what types of orbits could be found. It became evident that diminishing returns set in at much smaller values, and since $K$ represents the number of years the establishment process will take, an upper limit of 5 was set for each.

Each run through the genetic algorithm took an average of 3 minutes with a 2.53 GHz processor. Based upon the initial population, which were generated randomly, different trajectories were found. It is desirable to find trajectories with different K values, so eventually these values were constrained to be just one number with no variation. The genetic algorithm was then run at least 20 times (more if there was significant deviation in results) to find the other values that gave the lowest $\Delta \mathrm{V}$.

## C. STOUR Analysis

The establishment-cycler trajectory solutions found by the genetic algorithm in the circular, co-planar model can be verified by using the Satellite Tour Design Program (STOUR) developed by the Jet Propulsion Laboratory for the Galileo mission tour design. ${ }^{20}$ STOUR uses an analytical ephemeris for the location of the planets and the patched-conic method to calculate the gravity-assist trajectories.

Incorporating the $\mathrm{V}_{\infty}$-leveraging maneuver in STOUR was achieved with the $\Delta \mathrm{V}$-EGA subroutine developed by A. J. Staugler in 1996 at Purdue University. ${ }^{21}$ This subroutine allows the user to specify launch date, nominal orbit, approximate deviation in launch $\mathrm{V}_{\infty}$ from the nominal, location of the maneuver in terms of true anomaly, and number of heliocentric spacecraft and Earth revolutions between the maneuver and the Earth flyby. With the parameters of the $\mathrm{V}_{\infty}$-leveraging maneuver specified, STOUR then finds all possible patched-conic trajectories to the subsequent Mars encounter. To ensure that the desired cycler trajectory is found, launch dates were searched over a three-year period in order to encompass the $2 \frac{1}{7}$ year Earth-Mars synodic period. The starting launch date was chosen to be no sooner than 2022. For cycler trajectories with aphelion radii near that of the perihelion of Mars, search windows larger than three years were required, as the relative orientation between Earth and Mars must occur when Mars is near its perihelion.

Once STOUR calculates the trajectories for the specified span of launch dates, those trajectories that have orbit characteristics similar to the cycler trajectory of interest are identified. First, trajectories with the desired cycler shape are found by matching the aphelion and perihelion radii for the Earth to Mars arc to the theoretical aphelion and perihelion radii of the cycler (in the circular, co-planar model). The trajectory then must be checked to ensure the transfer occurs on the correct segment of the orbit. For example, on the Earth-to-Mars leg of the outbound Aldrin cycler, the spacecraft must encounter Earth just before perihelion on the post-Earth flyby orbit and arrive at Mars before reaching aphelion. The final trajectories chosen from the STOUR output over the three-year search were those that most closely matched the desired cycler orbit shape (i.e. aphelion and perihelion radii) while also providing the desired orbit segment on the trip from Earth to Mars for the first leg of the trajectory.

## III. Results

## A. Selected Cyclers and Their Characteristics

The trajectories considered for establishment are the Aldrin Cycler, ${ }^{9}$ the VISIT cyclers, ${ }^{10-12}$ and various two synodic period cyclers. ${ }^{13,14}$ The orbital elements of the various cycling trajectories are presented in Table 1. The VISIT-1 and VISIT-2 cycling trajectories are not specific trajectories, but a class. The $n\left(R_{p}\right) r \pm$ notation ${ }^{14}$ used specifies their exact characteristics. In this notation, the VISIT-1 and VISIT-2 cycling trajectories considered were $7(0.94) 12 \pm$ and $7(0.95) 10 \pm$, respectively. Several of the trajectories have multiple legs that change orbit characteristics after flybys of the Earth. However, to establish the cycler vehicle, only the characteristics of the first leg are used. The number of cycler vehicles is the amount needed to take advantage of each opportunity on both the outbound cycler (ascending or up escalator) and inbound cycler (descending cycler or down escalator). Each cycler trajectory passes through Earth orbit and Mars orbit atleast twice each, so it is crucial to know at which passage the spacecraft would encounter each planet. The Aldrin and Case 1 cyclers launch from Earth before perihelion, and all of the cyclers, with the exception of Case 1, encounter Mars before aphelion.

To get an initial idea of how these different cyclers compare, an impulsive $\Delta \mathrm{V}$ to get directly onto each is calculated. We assume that the Earth and Mars orbits are circular and co-planar and the initial orbit around Earth is circular at an altitude of 300 km . The values are shown in Table 2

## B. Genetic Algorithm Results

A subset of the establishment-cycler trajectories found from running the genetic algorithm are shown in Table 3. The solutions shown have a maximum K value of 5 , corresponding to $\mathrm{V}_{\infty}$ leveraging times of flight slightly less than 5 years. Longer $\mathrm{V}_{\infty}$-leveraging trajectories (corresponding to larger K ) were found, but diminishing returns of smaller overall $\Delta \mathrm{V}$ made them less attractive because of their longer flight times. The tolerances on the semi-major axis and eccentricity were set at 0.09 AU and 0.009 , respectively.

Table 1. Orbital elements and number of vehicles for selected cycler trajectories
\(\left.$$
\begin{array}{llllll}\hline \hline \text { Type } & \begin{array}{l}\text { Number } \\
\text { of Vehicles }\end{array} & \begin{array}{l}\text { Semi-Major } \\
\text { Axis, AU }\end{array} & & \text { Eccentricity }\end{array}
$$ \begin{array}{l}Aphelion <br>

Radius, AU\end{array}\right)\)| Perihelion |
| :--- |
| Radius, AU |

${ }^{\text {a }}$ These cyclers repeat every 7 Earth-Mars synodic periods, which usually means that 14 vehicles are needed. However, the VISIT cyclers encounter Earth and Mars more often than once every 15 years, so fewer vehicles are needed. ${ }^{14}$

Table 2. Direct $\Delta \mathrm{V}$ required to go from LEO to orbit with one impulsive burn for selected cyclers

| Type | Direct $\Delta \mathrm{V}^{\mathrm{a}}, \mathrm{km} / \mathrm{s}$ |
| :---: | :---: |
| Aldrin | 5.026 |
| VISIT-1 | 4.118 |
| VISIT-2 | 4.564 |
| Case 1 | 4.524 |
| Case 2 | 3.971 |
| Case 3 | 4.495 |
| S1L1 | 4.208 |
| U0L1 | 8.052 |

${ }^{\text {a }}$ Direct $\Delta V$ refers to launching directly from low Earth orbit (LEO) into the cycler trajectory without using any gravity assist maneuvers.

As an example, in Table 3 the Aldrin $4: 3(2)^{-}$launches from the Earth with a $\mathrm{V}_{\infty}$ of $2.638 \mathrm{~km} / \mathrm{s}$ on an orbit slightly more energetic than a $4: 3$ resonance orbit with the Earth. On the spacecraft's second orbit about the Sun, when only one orbit is fully completed, a $0.604 \mathrm{~km} / \mathrm{s}$ retrograde maneuver is performed to lower the perihelion of the orbit. The spacecraft completes this orbit and nearly completes an additional orbit, where before periapse on this last orbit around the Sun, the cycler vehicle intercepts the Earth with a $\mathrm{V}_{\infty}$ of $5.717 \mathrm{~km} / \mathrm{s}$. Then a $300-\mathrm{km}$ flyby changes the spacecraft's heliocentric orbit to have a semi-major axis of 1.61 AU and an eccentricity of 0.384 , which takes the cycler vehicle to Mars. The total $\Delta \mathrm{V}$ to achieve this final orbit is $4.118 \mathrm{~km} / \mathrm{s}$, a savings of $0.488 \mathrm{~km} / \mathrm{s}$ over the $4.606 \mathrm{~km} / \mathrm{s}$ required if the cycler were launched directly into the final orbit.

## C. STOUR Results

STOUR trajectory data for the given establishment-cycler trajectories are shown in Table 4. According to NASA's Mars Design Reference Architecture, ${ }^{22}$ the Earth-to-Mars time of flight should be 180 days or less for the health of the crew. The results from STOUR confirm that the Aldrin, Case 2, and U0L1 cyclers have times of flight that satisfy this requirement. The 4:3(2) ${ }^{-}$Case 3 and 4:3(2) ${ }^{-}$S1L1 establishment-cycler trajectories have the smallest Earth launch $\mathrm{V}_{\infty}$ values, as well as some of the smallest maneuver $\Delta \mathrm{V}$ values. The lowest maneuver $\Delta \mathrm{V}$ values come from the $4: 3(2)^{-}$VISIT-1 and 4:3(2)- Case 2 establishment-cycler trajectories, $0.043 \mathrm{~km} / \mathrm{s}$ and $0.059 \mathrm{~km} / \mathrm{s}$, respectively. Unfortunately, the VISIT-1 cycler requires more than 180 days to reach Mars, and, additionally, requires an infeasibly large number of cycler vehicles (up to 14 for missions that return the crew back to Earth). The 4:3(2)- Case 2 establishment-cycler trajectory, on

Table 3. Genetic algorithm results for the selected cyclers in the circular, co-planar model

| Type | K:L(M) | $\begin{gathered} \mathrm{V}_{\infty, \text { launch }}, \\ \mathrm{km} / \mathrm{s} \end{gathered}$ | $\begin{gathered} \Delta \mathrm{V}_{D S M}{ }^{\mathrm{a}} \\ \mathrm{~km} / \mathrm{s} \end{gathered}$ | $\begin{gathered} \mathrm{V}_{\infty, f l y b y} \\ \mathrm{~km} / \mathrm{s} \end{gathered}$ | $\begin{gathered} \mathrm{h}_{\text {flyby }}, \\ \mathrm{km} \end{gathered}$ | $\begin{gathered} \Delta \mathrm{V}_{\text {total }}, \\ \mathrm{km} / \mathrm{s} \end{gathered}$ | $\begin{aligned} & \mathrm{a}, \\ & \mathrm{AU} \end{aligned}$ | e | $\begin{gathered} \Delta \mathrm{V}_{\text {direct }} \mathrm{b} \\ \mathrm{~km} / \mathrm{s} \end{gathered}$ | $\Delta V$ Savings, km/s |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Aldrin | 4:3(2) ${ }^{-}$ | 2.638 | 0.604 | 5.717 | 300 | 4.118 | 1.61 | 0.384 | 4.606 | 0.488 |
|  | $3: 2(1)^{-}$ | 3.506 | 0.520 | 6.578 | 644 | 4.268 | 1.57 | 0.384 | 5.028 | 0.759 |
| VISIT-1 | 5:4(3) ${ }^{-}$ | 2.009 | 0.099 | 2.664 | 1175 | 3.482 | 1.22 | 0.184 | 3.520 | 0.038 |
|  | 4:3(2) ${ }^{-}$ | 2.503 | 0.047 | 2.848 | 644 | 3.530 | 1.22 | 0.184 | 3.565 | 0.035 |
| VISIT-2 | 5:4(3) ${ }^{-}$ | 2.052 | 0.310 | 3.754 | 300 | 3.701 | 1.36 | 0.266 | 3.827 | 0.126 |
|  | 4:3(2) ${ }^{-}$ | 2.550 | 0.233 | 3.993 | 1493 | 3.726 | 1.35 | 0.266 | 3.907 | 0.181 |
|  | $3: 2(2)^{-}$ | 3.341 | 0.181 | 4.611 | 1818 | 3.880 | 1.33 | 0.266 | 4.133 | 0.253 |
| Case 1 | 5:4(3) ${ }^{-}$ | 2.033 | 0.215 | 3.300 | 2179 | 3.602 | 1.29 | 0.229 | 3.688 | 0.085 |
|  | 4:3(2) ${ }^{-}$ | 2.523 | 0.126 | 3.378 | 338 | 3.614 | 1.29 | 0.229 | 3.711 | 0.097 |
|  | $3: 2(2)^{-}$ | 3.340 | 0.160 | 4.479 | 4160 | 3.859 | 1.26 | 0.233 | 4.083 | 0.224 |
| Case 2 | $5: 4(3)^{-}$ | 2.013 | 0.116 | 2.767 | 422 | 3.500 | 1.24 | 0.193 | 3.545 | 0.045 |
|  | $4: 3(2)^{-}$ | 2.507 | 0.064 | 2.968 | 1056 | 3.548 | 1.23 | 0.193 | 3.596 | 0.048 |
| Case 3 | 4:3(2) ${ }^{-}$ | 2.541 | 0.196 | 3.794 | 306 | 3.688 | 1.35 | 0.259 | 3.840 | 0.152 |
|  | $3: 2(2)^{-}$ | 3.341 | 0.180 | 4.607 | 2390 | 3.880 | 1.32 | 0.259 | 4.132 | 0.252 |
| S1L1 | 5:4(3) ${ }^{-}$ | 2.042 | 0.258 | 3.514 | 419 | 3.647 | 1.33 | 0.248 | 3.751 | 0.104 |
|  | 4:3(2) ${ }^{-}$ | 2.539 | 0.190 | 3.760 | 309 | 3.660 | 1.33 | 0.248 | 3.829 | 0.169 |
|  | $3: 2(2)^{-}$ | 3.340 | 0.160 | 4.479 | 2212 | 3.859 | 1.31 | 0.253 | 4.083 | 0.224 |
| U0L1 | 4:3(3) ${ }^{-}$ | 3.188 | 1.586 | 9.801 | 722 | 5.241 | 2.11 | 0.554 | 6.952 | 1.711 |
|  | $3: 2(1)^{-}$ | 3.752 | 1.417 | 10.362 | 300 | 5.244 | 2.07 | 0.554 | 7.333 | 2.089 |
|  | $2: 1(1)^{-}$ | 5.148 | 1.320 | 12.634 | 300 | 5.672 | 1.96 | 0.565 | 8.977 | 3.305 |

${ }^{\text {a }}$ Deep space maneuver that is performed during $\mathrm{V}_{\infty}$ leveraging at the aphelion.
${ }^{\mathrm{b}}$ Note that these Direct $\Delta V$ vary slightly from the values in Table 2. These values again assume the spacecraft is launching from a $300-\mathrm{km}$ circular orbit, but use the slightly different orbit characteristics found with the genetic algorithm. The tolerances on the semi-major axis and eccentricity are 0.09 AU and 0.009 , respectively.
the other hand, has an Earth launch $\mathrm{V}_{\infty}$ of only $2.522 \mathrm{~km} / \mathrm{s}$, and has an Earth-Mars flight time of 171 days. Another notable establishment-cycler trajectory is the 4:3(2)- Aldrin cycler, which has a time of flight of 161 days, an Earth launch $\mathrm{V}_{\infty}$ of $2.608 \mathrm{~km} / \mathrm{s}$, and a maneuver $\Delta \mathrm{V}$ of $0.568 \mathrm{~km} / \mathrm{s}$. It should also be noted that for all of the trajectories considered in Table 4, the closest approach flyby altitude is well beyond the safe altitude of 300 km .

The savings from $\mathrm{V}_{\infty}$ leveraging, in terms of both launch $\mathrm{V}_{\infty}$ and overall $\Delta \mathrm{V}$, are given in Table 5. The $3: 2(1)^{-}$Aldrin establishment-cycler trajectory requires a launch $\mathrm{V}_{\infty}$ of $6.546 \mathrm{~km} / \mathrm{s}$ if a direct launch into the orbit is performed, but with $\mathrm{V}_{\infty}$ leveraging, only $3.449 \mathrm{~km} / \mathrm{s}$ is needed, a savings of $3.097 \mathrm{~km} / \mathrm{s}$ (as indicated in the table). Taking into account the deep space maneuver, the $\Delta \mathrm{V}$ savings for this trajectory is $0.750 \mathrm{~km} / \mathrm{s}$.

When the establishment-cycler trajectories given in Table 4 were determined, the aphelion of the cycler trajectories was usually the first parameter examined to find a match to a particular cycler trajectory (out of the many trajectories found in STOUR). Once a match was found for the aphelion (usually to within a hundredth of an AU), the cycler trajectory with the closest to the desired perihelion was chosen as the candidate match. Because of this selection order (aphelion first, perihelion second), the aphelion values tend to match more closely to those determined by the circular, co-planar model than do the perihelion values, as observed in Table 4 and Table 1.

Two additional analyses further verify that the $4: 3(2)^{-}$Aldrin trajectory is indeed the trajectory desired. First, the $\mathrm{V}_{\infty}$ leveraging results from the STOUR trajectory were compared to those of Sims et al. ${ }^{19}$ They calculated the amount of $\mathrm{V}_{\infty}$ increase (from Earth launch to Earth flyby) expected for a given amount of maneuver $\Delta \mathrm{V}$. In Table 4 , the $\mathrm{V}_{\infty}$ increase for the $4: 3(2)^{-}$Aldrin cycler was about $2.90 \mathrm{~km} / \mathrm{s}$ due to a maneuver size of $0.57 \mathrm{~km} / \mathrm{s}$, which agrees with the increase predicted by Sims et al. of about $3 \mathrm{~km} / \mathrm{s}$ for the

Table 4. STOUR results for the selected cyclers in the analytic ephemeris

| Type | K:L(M) | $\begin{aligned} & \mathrm{LD}, \\ & \mathrm{~mm} / \mathrm{dd} / \text { yyyy } \end{aligned}$ | $\begin{array}{r} \text { TOF }{ }^{\text {a }} \\ \text { days } \end{array}$ | Periapse, AU | Apoapse, AU | $\begin{aligned} & \mathrm{V}_{\infty, \text { launch }}, \\ & \mathrm{km} / \mathrm{s} \end{aligned}$ | $\begin{aligned} & \Delta \mathrm{V}_{D S M}, \\ & \mathrm{~km} / \mathrm{s} \end{aligned}$ | $\begin{aligned} & \mathrm{V}_{\infty, f l y b y}, \\ & \mathrm{~km} / \mathrm{s} \end{aligned}$ | $\begin{array}{r} \mathrm{h}_{\text {flyby }}, \\ \mathrm{km} \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Aldrin | 4:3(2) ${ }^{-}$ | 01/03/2023 | 161 | 0.983 | 2.229 | 2.608 | 0.568 | 5.509 | 10,483 |
|  | $3: 2(1)^{-}$ | 12/09/2023 | 172 | 0.964 | 2.229 | 3.449 | 0.530 | 6.546 | 20,745 |
| VISIT-1 | 4:3(2) ${ }^{-}$ | 06/12/2046 | 201 | 1.012 | 1.470 | 2.540 | 0.043 | 2.834 | 120,625 |
| VISIT-2 | 4:3(2) ${ }^{-}$ | 12/27/2022 | 207 | 0.985 | 1.670 | 2.503 | 0.221 | 3.878 | 7,696 |
| Case 1 | 4:3(2) ${ }^{-}$ | 12/14/2024 | 365 | 0.946 | 1.503 | 2.558 | 0.346 | 4.526 | 140,286 |
| Case 2 | 4:3(2) ${ }^{-}$ | 04/22/2029 | 171 | 0.997 | 1.433 | 2.522 | 0.059 | 2.910 | 56,068 |
| Case 3 | 4:3(2) ${ }^{-}$ | 12/20/2022 | 216 | 0.988 | 1.651 | 2.492 | 0.187 | 3.688 | 13,291 |
| S1L1 | 4:3(2) ${ }^{-}$ | 12/20/2022 | 221 | 0.987 | 1.635 | 2.492 | 0.182 | 3.657 | 12,965 |
| U0L1 | $3: 2(1)^{-}$ | 11/25/2025 | 97 | 0.960 | 3.189 | 3.702 | 0.794 | 8.049 | 6,633 |

${ }^{\text {a }}$ Time of flight from Earth flyby to Mars.

Table 5. Savings accrued by using $\mathrm{V}_{\infty}$ leveraging

| Type | K:L(M) | $\begin{gathered} \mathrm{V}_{\infty} \text { Savings, } \\ \mathrm{km} / \mathrm{s} \end{gathered}$ | $\Delta \mathrm{V}_{\text {total }}, \mathrm{km} / \mathrm{s}$ | $\begin{aligned} & \Delta \mathrm{V}_{\text {direct }}, \\ & \mathrm{km} / \mathrm{s} \end{aligned}$ | $\begin{aligned} & \hline \Delta \text { V Savings }{ }^{\text {b }} \\ & \mathrm{km} / \mathrm{s} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Aldrin | 4:3(2) ${ }^{-}$ | 2.901 | 4.075 | 4.510 | 0.435 |
|  | $3: 2(1)^{-}$ | 3.097 | 4.261 | 5.011 | 0.750 |
| VISIT-1 | $4: 3(2)^{-}$ | 0.295 | 3.535 | 3.562 | 0.027 |
| VISIT-2 | $4: 3(2)^{-}$ | 1.375 | 3.704 | 3.868 | 0.164 |
| Case 1 | $4: 3(2)^{-}$ | 1.968 | 3.851 | 4.124 | 0.274 |
| Case 2 | $4: 3(2)^{-}$ | 0.388 | 3.546 | 3.581 | 0.035 |
| Case 3 | $4: 3(2)^{-}$ | 1.196 | 3.668 | 3.806 | 0.138 |
| S1L1 | 4:3(2) ${ }^{-}$ | 1.165 | 3.668 | 3.796 | 0.128 |
| U0L1 | $3: 2(1)^{-}$ | 4.347 | 4.604 | 5.845 | 1.241 |

${ }^{\text {a }}$ The difference between $\mathrm{V}_{\infty}$ flyby and $\mathrm{V}_{\infty}$ launch.
${ }^{\mathrm{b}}$ The difference between direct $\Delta \mathrm{V}$ and $\mathrm{V}_{\infty}$-leveraging total $\Delta \mathrm{V}$.
same sized $0.57 \mathrm{~km} / \mathrm{s}$ maneuver.
The second additional verification test on the $4: 3(2)^{-}$Aldrin trajectory was to determine whether the second leg of the cycler trajectory could be found. After departing Mars, this second leg must have the same orbit shape as the first leg, pass through the aphelion of the orbit, and then pass through the perihelion before arriving at Earth. Upon further simulations in STOUR, the second leg of the Aldrin cycler was found. Interestingly, this second leg had perihelion and aphelion values that more closely agreed with the theoretical values for the Aldrin cycler (as determined using a circular, co-planar model) than those of the first leg. A detailed description of the $4: 3(2)^{-}$Aldrin trajectory schematic is presented in Fig. 2. The $3: 2(1)^{-}$Aldrin trajectory was similarly verified.

## IV. Discussion of Results

## A. Genetic Algorithm

The resulting trajectories after the Earth flyby from the genetic algorithm matched fairly closely to the ideal characteristics shown in Table 1, but were not exact. They could have been made to match more closely by adjusting the tolerances set on the orbital elements, but this was not done for a couple of reasons. First, it would have taken more computer resources to run the algorithm to get a feasible solution. Also, the algorithm used the circular, co-planar model, so even if the trajectories found matched perfectly, there


Figure 2. The 4:3(2)- Aldrin establishment-cycler trajectory. The spacecraft launches (1) from the Earth onto the dashed orbit and completes one and a half orbits. At aphelion (2) of this orbit, a maneuver is performed to lower the perihelion and get onto the dotted orbit. The spacecraft completes one full orbit and returns to aphelion. The spacecraft continues inbound toward perihelion and flies (3) by the Earth, performing a gravity-assist maneuver. This maneuver raises the heliocentric energy of the orbit and puts the vehicle on the cycler trajectory (the solid line). Mars is encountered (4), but does not significantly affect the trajectory. The spacecraft completes its orbit and encounters (5) the Earth once more in order to continue on the cycler trajectory.
would still be error between them and the orbits that STOUR produced. The genetic algorithm was only meant to provide good guesses to STOUR, and it served its purpose well.

## B. STOUR

The trajectories found with STOUR have first leg cycler trajectory orbital parameters that closely match the theoretical values in the circular, co-planar model. However, there is no guarantee that these trajectories will continue to follow their respective cyclers on subsequent legs because small deviations could propagate into larger errors as the cycler progressed. In order to truly determine whether or not these first leg trajectories are on a given cycler, several subsequent legs would need to be simulated. However, because STOUR showed that the Aldrin cycler trajectories were capable of continuing onto their second legs, it is suspected that the other cycler trajectories found in STOUR will also be able to continue onto their subsequent cycler legs.

## C. Mass Savings

The $\Delta V$ savings of the STOUR results can be converted to mass savings with a few assumptions and the rocket equation. Following Nock ${ }^{23,24}$ the dry mass of a reasonable cycler vehicle is 70 mt . Nock's cycler was assumed to have low-thrust ion engines with an $\mathrm{I}_{s p}$ of 5000 s . However, the maneuvers in this study were done instantaneously, so a smaller $\mathrm{I}_{s p}$ needs to be assumed. For the maneuvers performed at Earth, an $\mathrm{I}_{s p}$ of 450 s was used, corresponding to a chemical rocket using liquid oxygen, liquid hydrogen. Due to boil off, this type of engine cannot be used for the deep space maneuvers, which can occur years after launch. For this maneuver, something comparable to the Galileo spacecraft was assumed, a monomethylhydrazine (MMH) and mixed oxygen and nitrogen (MON) thruster, which has a specific impulse of 300 s .

The propellant masses required were found using the rocket equation and Table 6 shows the results. It should be noted that for the $\mathrm{V}_{\infty}$-leveraging case, first the propellant required for the deep space maneuver is calculated, then that mass is added to the cycler mass to provide the initial mass for the launch maneuver.

The propellant requirements for some of the cyclers is significantly reduced, while others are only slightly improved. For example, the propellant needed for the 3:2(1)- Aldrin Cycler is reduced by 22.7 mt , or $15 \%$.

Table 6. Propellant mass savings of the various cyclers from $V_{\infty}$ leveraging

| Type | $\mathrm{K}: \mathrm{L}(\mathrm{M})$ | Launch $\mathrm{V}_{\infty}$ <br> Savings, $\mathrm{km} / \mathrm{s}$ | $\Delta \mathrm{V}$ Savings, <br> $\mathrm{km} / \mathrm{s}$ | Direct $\Delta \mathrm{V}$ <br> Prop., mt | $\mathrm{V}_{\infty}$Leveraging <br> Prop., mt | Prop. Mass <br> Savings, mt |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Aldrin Cycler | $4: 3(2)^{-}$ | 2.901 | 0.435 | 124.5 | 118.0 | 6.5 |
|  | $3: 2(1)^{-}$ | 3.097 | 0.750 | 147.9 | 125.2 | 22.7 |
| VISIT-1 | $4: 3(2)^{-}$ | 0.295 | 0.027 | 86.9 | 86.7 | 0.2 |
| VISIT-2 | $4: 3(2)^{-}$ | 1.375 | 0.164 | 98.2 | 96.1 | $2.0^{\mathrm{a}}$ |
| Case 1 | $4: 3(2)^{-}$ | 2.030 | 0.274 | 108.2 | 104.4 | 3.8 |
| Case 2 | $4: 3(2)^{-}$ | 0.388 | 0.035 | 87.6 | 87.4 | 0.2 |
| Case 3 | $4: 3(2)^{-}$ | 1.196 | 0.138 | 95.8 | 94.2 | $1.7^{\mathrm{a}}$ |
| S1L1 | $4: 3(2)^{-}$ | 1.165 | 0.128 | 95.4 | 94.1 | 1.3 |
| U0L1 | $3: 2(1)^{-}$ | 4.347 | 1.241 | 193.2 | 147.4 | 45.8 |

${ }^{\text {a }}$ These values avoid roundoff error by including more significant figures than displayed in the table.

On the other hand, the $3: 2(1)^{-}$VISIT-1 cycler reduces its propellant needs by only 0.2 mt , or less than $1 \%$. This small improvement does not seem to justify the extra approximate three years needed to establish the cycler.

## D. Low Thrust

For the analysis in this paper, impulsive $\Delta V$ are used for all the maneuvers. The only concern was to determine whether $\mathrm{V}_{\infty}$ leveraging was a worthwhile technique for Earth-Mars cycling trajectories and to determine how the different establishment-cycler trajectories compare to one another.

In a future work, a more practical (i.e. propellant-efficient) trajectory may be found by implementing low-thrust during the establishment phase, which would give a more complete picture of the mass savings that could be accrued by using $\mathrm{V}_{\infty}$ leveraging. If low-thrust is employed, it could also be used for the the establishment of powered cyclers, such as a one-synodic period powered cycler ${ }^{25}$ or the low-thrust Aldrin cycler. ${ }^{26}$

## E. Further Reflections

From this analysis it can be deduced that there is not one cycling trajectory that is always superior to the others. Rather the trajectory that should be chosen is highly mission dependent. For example, the current convention is that there should be a series of Mars missions where humans travel to Mars, stay for a given period of time, and then return safely to Earth. For this type of architecture, the low-thrust Aldrin cycler appears the most attractive because it uses only two vehicles to send and return humans at each opportunity.

There is another (albeit controversial) alternative for sustained human presence on Mars, i.e. a one-way trip. Such a commitment to permanently put astronauts on Mars, to live out the rest of their lives, would have three main benefits. First, the mission would be safer because the traditional Mars mission's Mars ascent and Earth descent phases, as well as the Mars-to-Earth interplanetary return voyage, would not be necessary. Second, this mission architecture would significantly reduce the overall monetary costs of sending humans to Mars. And finally, the colony would be immune to budget cuts after establishment as the crew would depend on enduring support from the Earth.

In the one-way to Mars scenario, the two-synodic period cyclers become the preferred method. The two-synodic period cyclers are generally less massive and have lower Earth $\mathrm{V}_{\infty}$ than the one-synodic period, low-thrust Aldrin cycler. Ordinarily, for the two-synodic period cyclers, if the crew were to return to the Earth, four cycler vehicles would be required (two inbound and two outbound if every opportunity is taken). However, a one-way trip would not require the inbound cycler vehicles, so only two vehicles would be required (or one if every opportunity were not taken). Of the two-synodic period cyclers analyzed in this work, the Case 2 cycler seems to be the most attractive. The Earth-to-Mars time of flight of 171 days is acceptable from NASA's human factors standpoint ${ }^{22}$ and it has a low overall propellant cost.

## V. Conclusion

The concept of cycler vehicles, which would provide safe havens for astronauts traveling to and from Mars on repeating trajectories, has been discussed since Hollister introduced the idea in 1969. Several examples of Earth-Mars cycler trajectories exist, including the Aldrin cycler, the VISIT's 1 and 2, and various twosynodic period cyclers. Though significant analyses have been devoted to ballistic and powered cyclers, no work has been presented on the problem of establishing a cycler orbit, i.e. transferring the cycler vehicle from LEO into the cycler trajectory. The present work makes preliminary inroads into the "establishment problem" for near-ballistic cyclers, assuming impulsive $\Delta \mathrm{V}$.

The $\mathrm{V}_{\infty}$-leveraging technique demonstrates that a significant Earth launch $\mathrm{V}_{\infty}$ reduction can be achieved in the establishment of the Aldrin cycler, the VISIT-2 cycler, and two-synodic period cyclers. The original concept of a cycler trajectory was based upon the idea that after a large investment was placed into the establishment of the cycler trajectory, there would be significant returns on the investment as the cycler vehicle is used over and over again. In addition, the cycler concept provides a comfortable and safe environment for the astronauts.

The preliminary results presented here suggest a significant reduction in propellant mass for establishing some of the cycler trajectories. For example, the establishment of an Aldrin cycler using the $3: 2(1)^{-} \mathrm{V}_{\infty^{-}}$ leveraging technique could potentially reduce the propellant mass (compared to an impulsive injection into the cycler trajectory by an impulsive burn from LEO) by 22.7 mt . These findings presents another advantage in the efficacy of the cycler concept.

Humans will one day live on Mars. The year 2019 will mark the $50^{\text {th }}$ anniversary of the first human landing on the Moon. As Buzz Aldrin has suggested, that historic anniversary would present an ideal time for a future President to announce a commitment, similar to that of President Kennedy's, to establish a permanent human presence on the planet Mars within the following two decades.

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