Fundamentals of Wind Energy Conversion for Electrical Engineers

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Abstract. These notes present the main technologies used today for converting wind energy to electrical energy. They are meant to be used as a supplement to introductory junior-level courses in electric power systems and/or senior-level electric machines and power electronics courses. Several textbooks (e.g., [**6**]) contain a very good fundamental treatment of electric machines, including basic design principles, which is not going to be reproduced herein. Also, basic knowledge of three-phase ac circuits and their steady-state analysis using phasors is a prerequisite. Herein, we discuss the details of generating electric energy from wind, and we present methods to analyze the most common wind energy conversion topologies. The "steady-state" of the wind energy conversion process is emphasized. Quotation marks are used because wind turbines are never in a steady-state due to the constant fluctuations of wind; however, they can be assumed to be in some kind of quasi-steady state because the wind variations are typically slower than the electrical dynamics of interest. The design of wind turbines is a multi-disciplinary project, and good designs are products of healthy collaboration of teams of engineers. But as electrical engineers, the problem we are mainly interested in is the analysis of the "electrical part" of a wind turbine. Other engineering disciplines are involved with the design and manufacturing of the remaining components, such as the nacelle, the blades, the hub, the gearbox, the tower, the foundations, as well as with supply chain and transportation issues, or the layout of wind power plants. Our domain of interest essentially consists of the generator, any associated power electronics converters (if they exist), and the electrical system that it is connected to.

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CHAPTER 1

Energy in the Wind Stream

1.1. Basic calculations

The objective of this section is to calculate the amount of kinetic energy in the wind stream, and how much of that can be practically harnessed. This will be the mechanical input on the shaft of our generators. To simplify things, we will do this by considering an "ideal" kind of wind and some elementary principles from physics. Our ideal wind flows in a single horizontal direction only, without any turbulence whatsoever (this is called an *irrotational* flow). The viscosity of the wind is also neglected (so the flow is called *inviscid*). The wind speed is denoted by v_w and is constant in time and space (we can call this a *steady* flow). The flow of the wind is also assumed to be of the *incompressible* kind, meaning that the density ρ of wind remains constant within a volume that moves with the same velocity as the wind. Moreover, let's assume that density is constant not only locally (as we track the motion of a small volume of air), but that all of the air has the same density. Note that the density of air depends on factors such as the temperature, the altitude, or the humidity. A typical value that is often used is $\rho = 1.25$ kg/m^3.

Now consider a vertical disc of radius R and surface area $A = \pi R^2$ through which the wind flows perpendicularly. Also let us visualize a cylindrical mass of air of exactly the right size so that it can pass snugly through the disc. If you thought of a thin cylinder, it would look like a coin balancing on its side, but it would not have any solid sides; it's just a cylinder of air. This cylinder of air has height h and volume $V = \pi R^2 h$. The kinetic energy of the wind mass m that is contained therein is $E = \frac{1}{2} m v_w^2 = \frac{1}{2} \rho \pi R^2 h v_w^2$. If somehow we were able to extract *all* this kinetic energy in an amount of time t, we would obtain an average power $P = E/t$. Since $h = v_w t$ (the wind speed was assumed to be constant), the time eventually disappears from the expression, which now becomes

$$P = \frac{1}{2} \rho \pi R^2 v_w^3 \,. \tag{1.1}$$

This is really convenient, because this expression works for any arbitrary time length t, so we can let t become infinitesimally small ($t \to 0$) and we can thus interpret this expression as a continuous rate of energy extraction from the wind. (Of course, this would be expressed in Joules per second, otherwise known as Watts.)

Example 1.1. Calculate the *power density* of wind for $v_w = 8$ m/s.

The wind's power density is defined as $P/(\pi R^2)$, which is equal to $\frac{1}{2} \rho v_w^3$. Substituting yields $(0.5)(1.25)(8^3) = 320$ W/m^2. Note that the relationship between power and wind speed is cubic. So if the wind speed were to double, the power density would increase by a factor of eight.

1.2. The Betz limit

Unfortunately, extracting all the energy from a wind stream at the above rate is not possible. To see why, consider what would happen if we actually (somehow) did this. In this case, we would be removing the entire kinetic energy from the wind, so that right after our disc the wind speed would have to be zero. However, this would disrupt the flow of the wind, because new masses of wind that would like to cross the disc would find it increasingly more difficult (or even impossible) to do so. The above thought experiment reveals that we cannot and should not try to extract all the energy from the wind stream, but only some portion of it.

Let us now try to obtain a better understanding of what happens in practice. First, let us think about what would happen to our cylindrical mass of air if its speed somehow changed. It turns out that the diameter of the cylinder changes to accommodate the change of speed (here we are implicitly assuming that the shape remains cylindrical). To see why, write $m = \rho V = \rho \pi R^2 h = \rho \pi R^2 v_w t$. If we divide the mass by time, we obtain a quantity that is called the *mass flow rate*, which is denoted by $\dot{m}$ and has the physical significance of how much mass passes through the disc in time t (how many kilograms per second). For our ideal wind stream, the law of *conservation of mass* implies that this rate is constant and equal to $\dot{m} = \rho \pi R^2 v_w$. Hence, the product $R^2 v_w$ must also remain constant (because of our original assumption of incompressible flow), which leads to the conclusion that the radius of the cylinder depends on the speed.

Since in a wind turbine we are extracting kinetic energy from the wind, we expect the wind speed to eventually decrease. Of course, we cannot build a device that would change the wind speed abruptly (as a step change) because that would require an infinite amount of force (due to Newton's second law of motion). Therefore, the wind speed must decrease gradually. The maximum wind speed $v_w(x_1) = v_1$ is found *upstream* from the wind turbine, whereas the minimum speed $v_w(x_2) = v_2$ is *downstream*. The situation is depicted in Fig. 1.1. In this analysis, we are not really interested exactly how far the upstream and downstream points are from each other. (But of course, this is something that designers of wind power plants take very seriously into account. The wind turbines must be placed adequately far from each other to reduce aerodynamic interactions.)

As the speed decreases, the radius of the wind-containing cylinder is increasing, and the wind flow is restricted within a tubular boundary, outside of which nothing can escape. Note that all vertical cross-sections at any horizontal position x are circles, where the wind speed component in the horizontal direction is the same; this is why we wrote the wind speed as a single function of horizontal position, $v_w(x)$. (Therefore, we are implicitly assuming that there is no variation of wind speed in the vertical direction. This is a somewhat problematic assumption, because the wind speed in reality tends to be higher as you move away from the ground due to a phenomenon called *wind shear*. Also, since the cylinder's radius is increasing, there must be a component of wind speed in the vertical direction as well. Nevertheless, to simplify things, we will keep going.) Somewhere in between the upstream and downstream locations is our *actuator disc*, which represents the actual energy extracting device. Let us denote the radius of the disc as $R(x_d) = R_d$. We do not specify how this device looks like at this point, but we are assuming that somehow we are able to build it. Also, we are considering a very thin device without blades

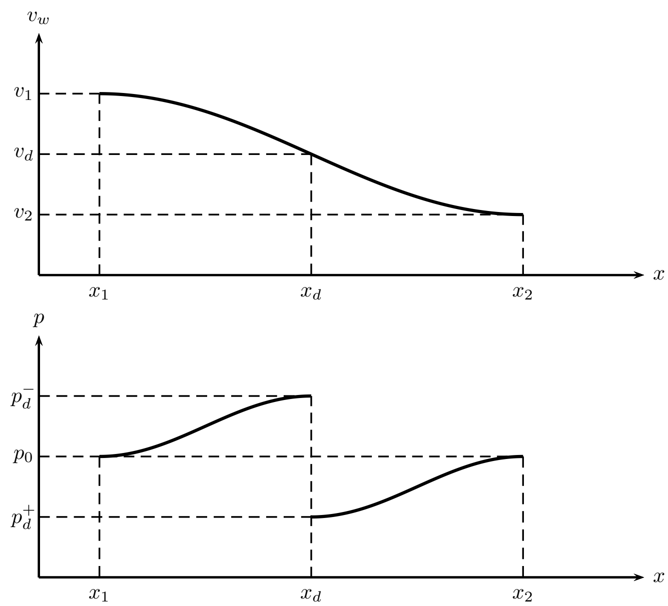

FIGURE 1.1. Variation of wind speed and static pressure from far upstream to far downstream.

so that the flow of the wind remains axial. Because of this abstraction, the ensuing analysis is valid for a wide variety of conceivable *horizontal-axis wind turbines* (also called HAWTs), under all the simplifying assumptions that were previously stated. The wind speed at the position of the disc is denoted by v_d. Obviously, $v_1 > v_d > v_2$.

Except from the tube's radius, another variable that changes together with wind speed is the *static pressure* p. Without going into too much detail, this phenomenon is explained by a law of physics called *Bernoulli's principle*, which states that the sum of static and *dynamic pressure*, $p+\frac{1}{2}\rho v_w^2$, must remain constant for a horizontal incompressible flow like we have assumed, but the constant is different for the flow before and after the disc. The pressure starts increasing from the value p_0 far upstream of the actuator disc (which represents the atmospheric pressure), until it reaches a maximum value p_d^- just before the disc. The disc introduces a step change in the static pressure, so that right after it the pressure drops to p_d^+, and keeps increasing until it reaches p_0 downstream. This pressure difference on both sides of the actuator disc can be thought to arise from a *drag* force $F = (p_d^- - p_d^+)\pi R_d^2$ that is pointing to the left, and which leads to a continuous extraction of power $P_d = F v_d$ by the disc (assuming that there are no losses). Applying Bernoulli's principle twice, we obtain $p_0 + \frac{1}{2}\rho v_1^2 = p_d^- + \frac{1}{2}\rho v_d^2$ for upstream and $p_0 + \frac{1}{2}\rho v_2^2 = p_d^+ + \frac{1}{2}\rho v_d^2$ for downstream, hence we can express the pressure difference as $p_d^- - p_d^+ = \frac{1}{2}\rho(v_1^2 - v_2^2)$. Therefore,

$$F = \frac{1}{2}\rho\pi R_d^2(v_1^2 - v_2^2)\,. \tag{1.2}$$

Now we introduce the law of *conservation of momentum*. This law is well known for a single body of mass m (i.e., $F = ma$), but now it must be applied to the entire flow stream. It turns out that the drag force from the actuator disc reduces the global momentum of the flow, and that this reduction can be related to the difference between the input and output *momentum rates* by

$$F = \dot{m}(v_1 - v_2)\,. \tag{1.3}$$

Choosing the expression of mass flow rate at the location of the disc, $\dot{m} = \rho\pi R_d^2 v_d$, substituting in (1.3) and equating this with (1.2) leads to

$$v_d = \frac{v_1 + v_2}{2}\,. \tag{1.4}$$

We have just proved an important result, namely, that the wind speed at the actuator disc is the average value of the upstream and downstream wind speeds. Using this result we can now calculate the maximum extraction of power from the wind stream.

More accurately, the problem we are trying to solve is the following: Given an upstream wind speed v_1, what is the downstream wind speed v_2 that leads to maximum power extraction from the actuator disc? To solve this, we rewrite the equation for power extraction as

$$P = Fv_d = \frac{1}{4}\rho\pi R_d^2 (v_1 - v_2)\,(v_1 + v_2)^2\,. \tag{1.5}$$

This is an equation of the form $P = P(v_2)$, and its maximum point is obtained by setting the derivative dP/dv_2 equal to zero. This yields

$$-(v_1 + v_2)^2 + 2(v_1 + v_2)(v_1 - v_2) = 0 \Rightarrow v_2 = \frac{v_1}{3}\,. \tag{1.6}$$

(To confirm that this point corresponds to a maximum one can evaluate the second derivative d^2P/dv_2^2, which should turn out to be negative.) This also leads to

$$v_d = \frac{2}{3}v_1\,. \tag{1.7}$$

The maximum possible power extraction from the wind stream is then

$$P_{\max} = \left(\frac{16}{27}\right)\left(\frac{1}{2}\rho\pi R_d^2 v_1^3\right)\,. \tag{1.8}$$

The factor $16/27 \approx 59\%$ is called the *maximum power coefficient*, and $P_{\max}$ is commonly called the *Betz limit*, named after the German physicist Albert Betz [**11**].

Example 1.2. Calculate the Betz limit for a horizontal-axis wind turbine whose blades sweep a circular area with a diameter of 80 m, and for an upstream wind speed of $v_1 = 12$ m/s.

The answer is found by substituting the given parameters in (1.8):

$$P_{\max} = \left(\frac{16}{27}\right)\left(\frac{1}{2}\right)(1.25)(3.14)(40^2)(12^3) = 3.2 \text{ MW}\,. \tag{1.9}$$

At this point, it is worthwhile to discuss briefly the interpretation of the Betz limit. It is often said (rather incorrectly) that this represents the maximum possible efficiency of a wind turbine. Remember that energy conversion efficiency is defined as $\eta = P_{\text{out}}/P_{\text{in}}$. However, a closer look at (1.8) reveals that the stuff inside the second parentheses is not representative of the input power, even though it resembles the expression (1.1)! In fact, it is not representative of the power flowing through any cross-section of the flow tube whatsoever. For example, the input power of the wind stream is $P_{\text{in}} = \frac{1}{2}\rho\pi R(x_1)^2 v_1^3$, where $R(x_1)^2 = (v_d/v_1)R_d^2 = (2/3)R_d^2$. So $P_{\text{in}} = \frac{2}{3}\frac{1}{2}\rho\pi R_d^2 v_1^3$, which yields a maximum efficiency

$$\eta_{\max} = \frac{\frac{16}{27}}{\frac{2}{3}} = \frac{8}{9} = 88.9\%\,. \tag{1.10}$$

So, when operating at the Betz limit, a wind turbine absorbs 88.9% of the available power in the flow tube! (I would not hesitate to call it a very efficient device.) An alternative interpretation of the 59% factor is the ratio of power extracted from the turbine over the power of the *unperturbed* wind stream that would exist at the same location before erecting the turbine. The reason why Betz originally wrote the equation in this form is because he was not that much interested in the real efficiency of the device, because of the abundance of wind energy in the atmosphere. Rather, he wanted to answer the following question, which has significant engineering design and economical consequences: "how much energy can be obtained in the most favourable case from a rotor of given diameter in a wind of given speed?" [**2**]

Of course, real wind turbines can never reach the Betz limit, which was obtained under several simplifying assumptions. In general, relaxing the assumptions that were made leads to increased losses and a subsequent reduction in performance. The real wind turbines tend to operate at lower "efficiencies," reaching as high as 40–45%. Aerodynamics engineers strive to optimally design the blades, hub, and nacelle, in order to reach as close to the Betz limit as possible.

1.3. Wind turbines' power characteristics

In practice, it can be shown using more advanced principles of aerodynamics that the power extracted by a wind turbine can be expressed as

$$P = \frac{1}{2}C_p(\lambda, \theta)\rho\pi R_d^2 v_1^3\,. \tag{1.11}$$

The function C_p is called the *performance coefficient*, and it depends on something called the *tip-speed ratio* λ and the blades' *pitch angle* θ. The tip-speed ratio is defined as

$$\lambda = \frac{\omega_w R_d}{v_1} \tag{1.12}$$

and it represents the ratio between the circumferential speed of the tips of the blades over the upstream wind speed. The parameter ω_w denotes the angular velocity of the blades, in rad/s. The explicit dependence on θ is due to the fact that modern wind turbines have the capability to alter the pitch of the blades by rotating them. Without going into too much detail, pitching the blades away from the optimal point where $\partial C_p/\partial\theta = 0$ in effect reduces the aerodynamic torque on the blades and the power extracted from the wind. This functionality is useful when a turbine needs to limit its power output, or when it needs to slow down.

A typical variation of C_p is shown in Fig. 1.2. Because of the shape of these curves, to harvest maximum power from the wind the turbine should operate at the peak that occurs at the optimal tip-speed ratio (this varies with blade pitch angle). Another interesting plot is the one shown in Fig. 1.3, which depicts the power extracted from the wind as a function of the blades' rotational speed, for

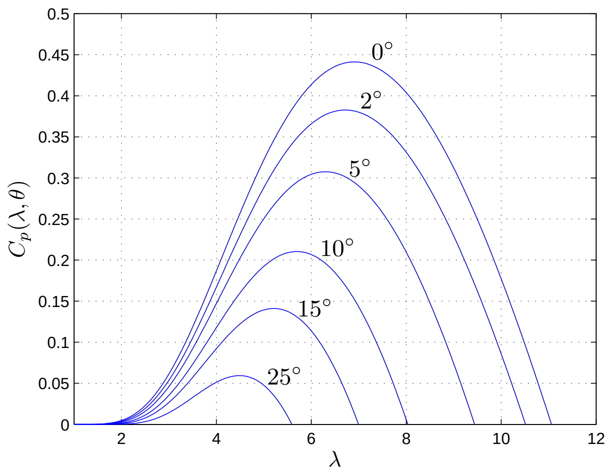

FIGURE 1.2. A typical performance coefficient variation for a range of blade pitch angles $\theta = \{0, 2, 5, 10, 15, 25\}°$.

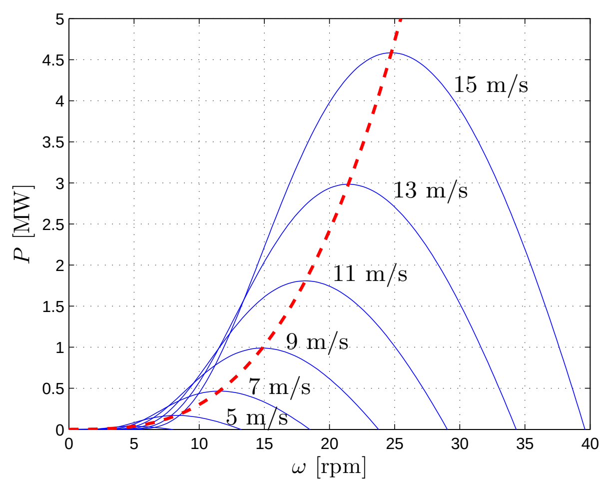

FIGURE 1.3. Turbine power vs. blade speed for wind speeds $v_1 = \{3, 5, \ldots, 15\}$ m/s, for a turbine with an 80-m wide swept area and $C_p(\lambda, 0)$ from Fig. 1.2.

different wind speeds (and assuming that the blades are not pitched). The red dashed line connects the peak points of all the power curves, and represents the maximum power extraction curve. This curve is cubic with respect to v_w and ω (because these two are proportional at the peaks), and so it rises very fast as the wind speed increases.

The most common wind turbines in the market today are rated for 1.5–2.5 MW, so something should be done to limit their output for high wind speeds (say, higher

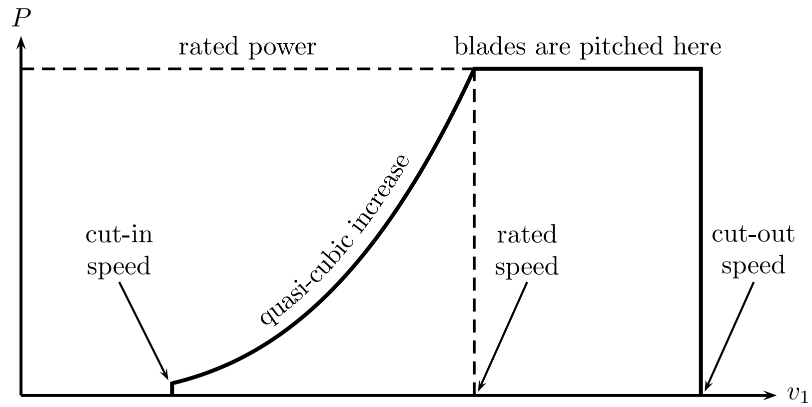

FIGURE 1.4. Wind turbine power output as a function of wind speed.

than 12 m/s). The wind speed at which a turbine reaches its nominal power output is called the *rated wind speed*. Above this value, pitching of the blades occurs, and the power output is maintained constant at rated value. (In practice, the power output is not exactly constant due to the fluctuations of wind speed, but it stays close to the rated value because of an automatic control system that adjusts the blade pitch angle to make this happen. Some wind turbines do not have active pitch control systems. These turbines are *stall-regulated*, meaning that their speed is restrained from reaching dangerously high values because their blades are designed to stall above a certain speed threshold.) The wind turbine stops producing power at a maximum wind speed called the *cut-out speed*. Also observe that for low speeds the power production is insignificant. For instance, the 3 m/s curve is almost indistinguishable in the plot, having a peak power around 35 kW. So there is a minimum wind speed, called the *cut-in speed*, below which a turbine just shuts itself down because there is not enough energy in the wind to supply its own mechanical losses and other auxiliary systems. A typical power production curve vs. wind speed of a variable-speed wind turbine is sketched in Fig. 1.4. Of course, in our analysis so far we have implicitly assumed that the turbines have *yaw control* systems to align with the prevailing wind direction. It should be noted that the output power is equal to the aerodynamic power that we have calculated minus all mechanical and electrical losses incurred from the blades to the electrical terminals.

EXAMPLE 1.3. For an upstream wind speed of $v_1 = 10$ m/s, what is the optimal angular velocity for a wind turbine with an 80-m wide swept area and the performance coefficient of Fig. 1.2?

Assuming that the blades are not pitched (i.e., normal operating mode with $\theta = 0°$), the optimal tip-speed ratio is $\lambda^* \approx 6.9$. The optimal angular velocity is then $\omega_w^* = \lambda^* v_1 / R_d = (6.9)(10)/(40) = 1.73$ rad/s or 16.5 rpm (revolutions per minute). This corresponds to about 4 seconds per revolution. Next time you drive by a wind farm on a windy day, try to count how many seconds it takes for each revolution and compare it with this number. On a less windy day, the turbines will rotate slower.

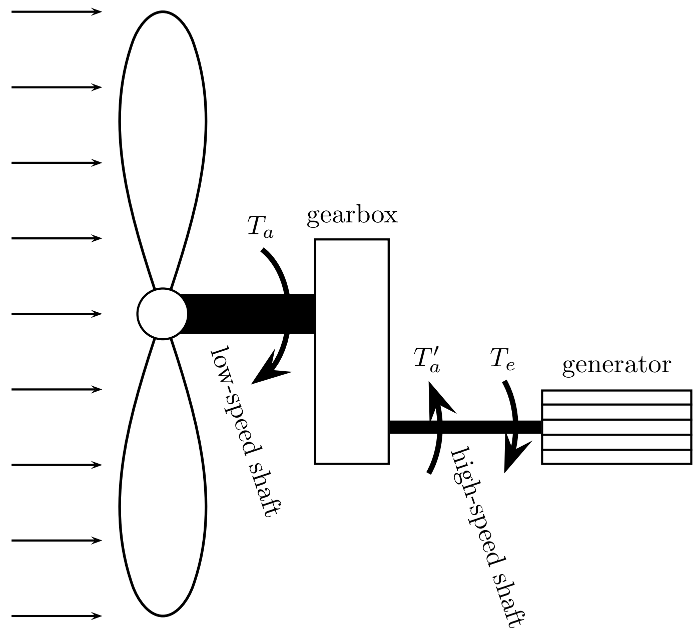

FIGURE 1.5. Schematic of a wind turbine drivetrain.

From the previous discussion we conclude that *for maximum power extraction the blades' rotational speed must be proportional to the prevailing wind speed*, which of course varies continuously. This objective is achieved in modern wind turbines by an automatic speed control system. Older wind turbines did not have this capability; they were essentially *fixed-speed machines* so they were not able to get the most out of the wind due to operation with sub-optimal tip-speed ratios. In modern turbines, which are of the *variable-speed* kind, the speed is controlled by adjusting the electromagnetic torque of the electrical generator (this acts in a direction opposite of the aerodynamic torque and tends to brake the blades). The technology of *power electronics and motor drives* enables us to control the electromagnetic torque with high precision. Usually, the electrical generator is connected to the blades with a gearbox. The blades are connected to a *low-speed shaft*, whereas the generator is connected to a *high-speed shaft*, as depicted in Fig. 1.5. The *aerodynamic torque* is reduced from a high value at the low-speed side T_a to a low value T_a' at the high-speed side; for an ideal lossless gearbox, these are related by the gearbox ratio $G > 1$: $T_a' = T_a/G$. The speed of the high-speed shaft can be determined by the simple equation of motion

$$J\frac{d}{dt}\omega_{rm} = T_a' - T_e \qquad (1.13)$$

where J is the combined moment of inertia of the blades, shafts, gearbox, and generator rotor, reflected to the high-speed side. By controlling the generator's electromagnetic torque T_e we can make the rotor accelerate or decelerate, to remain as close as possible to the optimum tip-speed ratio.

In general, the *annual energy yield* of a wind turbine depends on the variation of wind speed over a year and the generator's power curve. A generator will also be shut down periodically for maintenance. So a wind turbine does not provide its rated power output on a continuous basis. Rather, it provides on average an amount of power that is significantly lower than rated. The ratio of the turbine's average power over its rated capacity is called the *capacity factor*. This is typically on the order of 30-40% for MW-scale onshore turbines, and could be higher for offshore turbines (where the winds are typically better).

In the following chapters we will discuss in more detail the most common topologies for the conversion of the wind's mechanical energy to electrical energy.

1.4. Exercises

(1) A wind power plant has 100 turbines, each one rated for 1.5 MW. The capacity factor is 35%. What is the plant's annual energy yield?

(2) Clipper Windpower had announced (in 2007) its plans to produce the Britannia 7.5 MW offshore wind turbine, also called the MBE (Million Barrel Equivalent), which at the time would have been the world's largest turbine.
  - (a) Estimate the blade diameter.
  - (b) Estimate how many years it will take for the turbine to generate the same amount of energy that we can obtain by "burning" 1,000,000 oil barrels in a heat engine.

(3) The turbines in large wind power plants are typically positioned in a rectangular grid configuration, which is oriented to face the prevailing wind direction. The distance between turbines is measured in terms of their rotor diameter. In onshore plants, typical grid dimensions are 3–5 diameters apart perpendicular to the prevailing wind and 5–10 rotor diameters apart parallel to the prevailing wind. In offshore plants, turbines are placed 7–8 diameters apart, equally in both directions. This distance is selected to minimize the interference between turbines, and maximize the energy obtained. In this problem, we are interested to evaluate the *land usage* of wind farms. (Of course, offshore wind power does not use 'land' per se, but you get the point.)
  - (a) Estimate the *average power per unit of land area*, where 'average' implies an averaging process over time. Your answer should be: x W/m^2. This involves a few "back-of-the-envelope"-type calculations. Make all necessary assumptions that you need, but justify each one adequately. Provide two answers: one for onshore, and one for offshore wind power plants.
  - (b) Explain why land usage is more or less independent of turbine size.
  - (c) Compare your answer to the land usage of solar energy plants.

CHAPTER 2

Squirrel-Cage Induction Generators

Squirrel-cage induction generators are commonly used in the topology shown in Fig. 2.1, also called a Type-1 wind turbine. They are connected directly to the power system (i.e., without a frequency-controlling power electronics interface), so their stator has ac voltages and currents of system frequency (60 Hz in the USA). A gearbox is necessary to connect the low-speed shaft where the blades are connected (remember, this spins at only a few rpm) to the high-speed shaft at the generator side. The induction generator rotor spins at almost synchronous speed, which is 1800 rpm for a four-pole machine or 1200 rpm for a six-pole machine (in the USA). So this type of wind turbine is of the fixed-speed kind.

2.1. Analyzing the equivalent circuit

The analysis of the steady-state operation of a squirrel-cage induction generator is based on its equivalent circuit, shown in Fig. 2.2, which is provided here without derivation [**6**]. This is very similar to the T-equivalent circuit of a transformer, and it represents one phase of a symmetric Y-connected three-phase machine. The parameters of the equivalent circuit will be given to us, and all we usually have to do

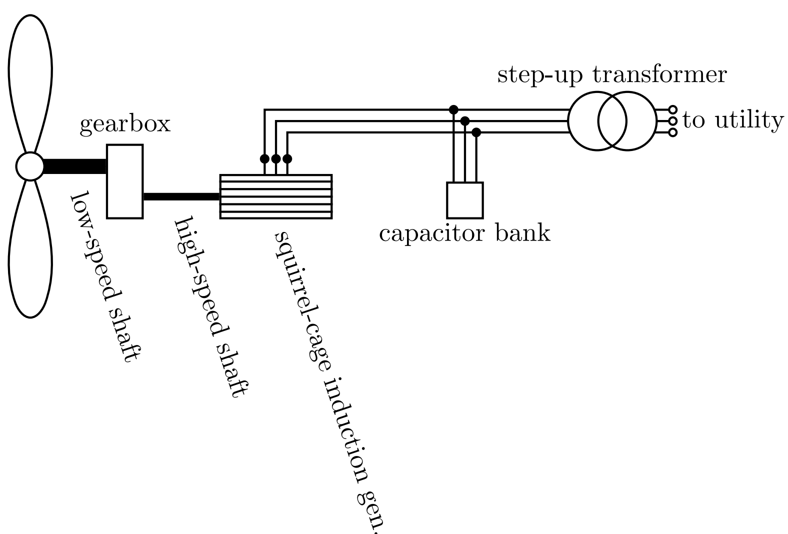

FIGURE 2.1. Topology of a Type-1 wind generator. (Note: A thyristor-based "soft-starter" power electronics converter is sometimes used, which is not shown here because it does not affect the steady-state operation. Also not shown is the parking brake.)

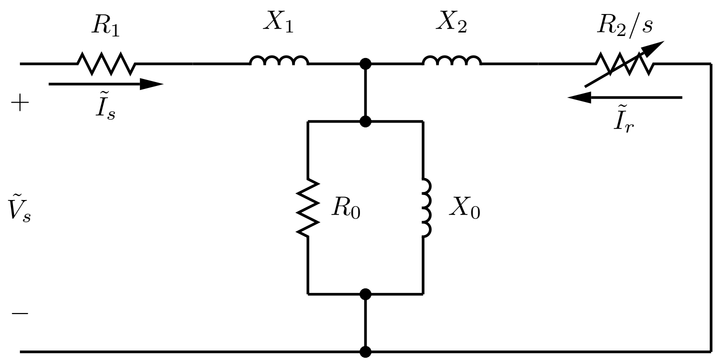

FIGURE 2.2. Equivalent steady-state circuit of the squirrel-cage induction machine.

is analyze the circuit for different operating conditions. The physical significance of the resistances and reactances of the equivalent circuit is similar to the transformer, and will not be discussed here further. However, there is one parameter that is new, namely s, which divides the resistance of the rotor R_2. This represents the machine's *slip*, which is defined as

$$s = \frac{\omega_e - \omega_r}{\omega_e}\,, \tag{2.1}$$

where $\omega_e = 2\pi f_e$ is the electrical system's angular frequency, and $\omega_r = \frac{P}{2}\omega_{rm}$ is the mechanical rotor speed multiplied by the number of *pole-pairs*, also called the *electrical rotor speed*. (Do not confuse this P with the symbol previously used for power; in this context it just denotes the number of magnetic poles of the machine, which is always an even number.) The slip is a way of expressing the relative difference in speed between the magnetic field arising from the stator currents and the mechanical speed of the rotor, or how fast the rotor "slips by" the stator field. In a squirrel-cage induction machine, the slip needs to be nonzero for anything interesting to happen. Otherwise, when the rotor is synchronized with the stator magnetic field, no currents are induced in the rotor windings, and no torque is generated. Remember that the stator electrical quantities (voltages and currents) have the frequency of the system (e.g., $f_e = 60$ Hz), whereas the induced electrical quantities in the rotor have *slip frequency*, sf_e.

The slip is a very important parameter, because it ties the mechanical side of things (how fast the rotor is spinning) with the electrical side. Without the slip, the equivalent circuit becomes identical to a transformer circuit, which is a motionless device. Note that the slip can take positive or negative values, depending on the sign of $\omega_e - \omega_r$. When the slip is positive, the machine becomes a motor; when it is negative the machine becomes a generator. One can observe that when the slip is negative, the equivalent circuit has a resistance with a negative value ($R_2/s < 0$) which, instead of consuming power, generates power. Of course, the actual rotor resistance does not change with speed; it is a constant that depends on the machine design. Sometimes the equivalent circuit is drawn slightly differently, with two resistances in series in the rotor side, $R_2/s = R_2 + \frac{1-s}{s}R_2$, to make this fact more explicit. When drawn like this, the physical significance of things becomes more obvious. The rotor circuits consume an amount of power equal to $3I_r^2R_2$ as ohmic loss. The other component of power, $3I_r^2\frac{1-s}{s}R_2$, arises from the process of *electromechanical energy conversion*, and represents the power going to or coming from the shaft. (There is no voltage or current source. The mechanical shaft power is consumed or produced by a resistance!)

EXAMPLE 2.1. A six-pole induction motor is connected to a 50-Hz system (somewhere in Europe). The slip is $s = 0.01$, so the machine acts as a motor.

- What is the speed of the rotor in rpm?
  The electrical rotor speed can be found from (2.1): $\omega_r = (1-s)\omega_e = (0.99)(2\pi)(50) = (2\pi)(49.5)$ rad/s, or 49.5 Hz. The actual rotor speed is found by taking into account the number of magnetic poles (six in this case): $f_{rm} = (49.5)/(3) = 16.5$ Hz, or 990 rpm.
- What is the frequency of the currents in the stator winding?
  The stator carries 50-Hz currents.
- What is the frequency of the currents in the rotor?
  The rotor carries currents of frequency $sf_e = (0.01)(50) = 0.5$ Hz. The magnetic field of the rotor also has this frequency, but because it is spinning (electrically) at 49.5 Hz, the rotor and stator magnetic fields are synchronized, and constant electromagnetic torque is produced.
- What happens if the slip becomes -0.01?
  This means that the machine is now acting as a generator. The rotor spins faster than the stator magnetic field, at 50.5 Hz electrical, or 1010 rpm. The stator still carries 50-Hz currents, but the rotor carries currents of frequency -0.5 Hz. A frequency with a negative sign means that the rotor's magnetic field spins at the opposite direction than before (relative to the rotor). Therefore, synchronism between the rotor and stator fields is maintained.

We proceed with an extended example of how the induction machine equivalent circuit can be solved for any given operating condition. Let us use the parameters of the 15-HP electric machine from p. 293 of Kirtley's textbook [**6**], which are repeated here for convenience: $P = 4$ (number of poles), $V_s = 138.6$ V line-to-neutral rms (240 V line-to-line), $f_e = 60$ Hz, $R_1 = 0.06\ \Omega$, $R_2 = 0.15\ \Omega$, $X_1 = 0.44\ \Omega$, $X_2 = 0.43\ \Omega$, $X_0 = 12.6\ \Omega$. No information is provided for R_0 so we will not use this. Let us also assume that the slip is given to us, $s = -0.03$, so the machine is acting as a generator. The following three methods are possible ways to solve the equivalent circuit. What we mean by "solve" is to determine the currents that flow in the stator and rotor windings, $\tilde{I}_s$ and $\tilde{I}_r$, given the stator voltage and the slip. Once we know the currents, we can then proceed to compute other quantities of interest, such as the *efficiency* or the *power factor*.

2.1.1. Solving the equivalent circuit – Method 1. This method first finds the circuit's *input impedance*, and then proceeds backwards inside the equivalent circuit. This is similar to the method of Section 13.2.1 in the textbook [**6**].

(1) Find the *air-gap impedance* Z_g by combining two parallel branches:

$$Z_g = jX_0 // (R_2/s + jX_2) = \frac{1}{\frac{1}{jX_0} + \frac{1}{R_2/s + jX_2}} \tag{2.2}$$

Note that $R_2/s + jX_2 = -5 + j0.43\ \Omega$, with a negative real part. For low slip values, the rotor-side impedance is dominated by the resistive term. After calculations, we obtain $Z_g = -4.08 + j1.98\ \Omega$.

(2) Add the stator's impedance to the parallel combination, to find the total input impedance:

$$\begin{aligned} Z_{\text{in}} &= R_1 + jX_1 + Z_g \\ &= 0.06 + j0.44 - 4.08 + j1.98 \\ &= -4.02 + j2.42\ \Omega \end{aligned} \tag{2.3}$$

(3) Find the stator current:

$$\begin{aligned} \tilde{I}_s &= \frac{\tilde{V}_s}{Z_{\text{in}}} \\ &= \frac{138.6\angle 0}{-4.02 + j2.42} = \frac{138.6\angle 0}{4.69\angle 148.9^\circ} \\ &= 29.6\angle -148.9^\circ\ \text{A} \end{aligned} \tag{2.4}$$

(4) Compute the internal voltage across the magnetizing branch:

$$\begin{aligned} \tilde{V}_0 &= \tilde{V}_s - \tilde{I}_s(R_1 + jX_1) \\ &= 138.6 - (29.6\angle -148.9^\circ)(0.06 + j0.44) \\ &= 133.9\angle 5.2^\circ\ \text{V} \end{aligned} \tag{2.5}$$

(5) Compute the rotor current (note the minus sign due to assumed current direction in Fig. 2.2):

$$\begin{aligned} \tilde{I}_r &= -\frac{\tilde{V}_0}{R_2/s + jX_2} \\ &= \cdots = 26.7\angle 10.1^\circ\ \text{A} \end{aligned} \tag{2.6}$$

Alternatively, the rotor current could have been obtained directly (without the need to compute $\tilde{V}_0$) with a current divider:

$$\tilde{I}_r = -\frac{jX_0}{jX_0 + jX_2 + \frac{R_2}{s}} \tilde{I}_s \tag{2.7}$$

2.1.2. Solving the equivalent circuit – Method 2. This method is based on applying KCL at the (top) node of the magnetizing branch.

(1) The sum of the currents flowing out of the node is zero,

$$\frac{\tilde{V}_0 - \tilde{V}_s}{R_1 + jX_1} + \frac{\tilde{V}_0}{jX_0} + \frac{\tilde{V}_0}{\frac{R_2}{s} + jX_2} = 0 \tag{2.8}$$

or by rearranging terms,

$$\tilde{V}_0 \left(\frac{1}{R_1 + jX_1} + \frac{1}{jX_0} + \frac{1}{\frac{R_2}{s} + jX_2} \right) = \frac{\tilde{V}_s}{R_1 + jX_1} \tag{2.9}$$

which can be solved for $\tilde{V}_0$ yielding the same answer as before.

(2) Compute the rotor current using (2.6).
(3) Compute the stator current:

$$\tilde{I}_s = \frac{\tilde{V}_s - \tilde{V}_0}{R_1 + jX_1} \tag{2.10}$$

2.1.3. Solving the equivalent circuit − Method 3. This method is based on finding the Thévenin equivalent seen from the rotor, and it is similar to the one described in Section 13.2.1.1 of the textbook [**6**]. Recall that this circuit is formed by a voltage source in series with an impedance. In this case, the voltage source is the open-circuit voltage right after the magnetizing branch (at the rotor side), and the impedance is the one seen from that point looking to the left when the stator terminals are short-circuited.

(1) The Thévenin voltage can be found with a voltage divider:

$$\tilde{V}_{\mathrm{Th}} = \frac{jX_0}{R_1 + jX_1 + jX_0}\tilde{V}_s \tag{2.11}$$

Using the numerical values provided, we obtain

$$\tilde{V}_{\mathrm{Th}} = 133.9\angle 0.3^\circ \text{ V}$$

(2) The Thévenin impedance is

$$\begin{aligned} Z_{\mathrm{Th}} &= \frac{1}{\frac{1}{R_1+jX_1} + \frac{1}{jX_0}} \\ &= 0.06 + j0.43\ \Omega \end{aligned} \tag{2.12}$$

(3) The rotor current is found by

$$\tilde{I}_r = -\frac{\tilde{V}_{\mathrm{Th}}}{Z_{\mathrm{Th}} + \frac{R_2}{s} + jX_2} \tag{2.13}$$

(4) Calculate the stator current by an "inverse" current divider (cf. (2.7)):

$$\tilde{I}_s = -\frac{jX_0 + jX_2 + \frac{R_2}{s}}{jX_0}\tilde{I}_r \tag{2.14}$$

The advantage of Method 3 lies in the fact that it allows the quick calculation of rotor current (in one step) for any given value of slip using (2.13), hence it is really useful when plotting rotor-side quantities that depend on the slip, such as the shaft power and torque.

2.1.4. Calculating power and torque. We have previously shown using the equivalent circuit that the *mechanical power* that flows to the shaft (i.e., from the electrical to the mechanical side) is given by

$$P_m = 3I_r^2\frac{1-s}{s}R_2\,. \tag{2.15}$$

In our case, this is equal to: $(3)(26.7^2)\left(-\frac{1.03}{0.03}\right)(0.15) = -11$ kW. The minus sign means that the power flows in the opposite direction (from the mechanical to the electrical side), which makes sense because this machine is acting as a generator. The actual *shaft power* (positive for motor action, negative for generator action) is denoted by P_{sh}. This is equal to P_m minus *friction and windage* losses, P_w. In this example, this could be, say, $P_w = 500$ W. The amount of friction and windage loss will be given, because it cannot be computed using the electrical equivalent circuit. Therefore,

$$P_m = P_w + P_{sh}\,, \tag{2.16}$$

and $P_{sh} = -11.5$ kW. A small amount of power is dissipated as heat on the rotor:

$$P_d = 3I_r^2 R_2 \tag{2.17}$$

equal to $(3)(26.7^2)(0.15) = 321$ W. The sum of P_m and P_d can be thought to pass through the air-gap from the stator side, and is called the *air-gap power*, P_{ag}. The equation that relates these three powers is

$$P_{ag} = P_d + P_m = 3I_r^2 \frac{R_2}{s}\,. \tag{2.18}$$

The air-gap power is about -10.7 kW in our example. There is an additional component of ohmic loss on the stator windings, called *armature dissipation*, P_a:

$$P_a = 3I_s^2 R_1\,. \tag{2.19}$$

This is equal to $(3)(29.6^2)(0.06) = 158$ W. The *stator power* is what enters the motor terminals:

$$P_s = P_a + P_{ag} = P_a + P_d + P_w + P_{sh} \tag{2.20}$$

which equals $158 + 321 + 500 - 11500 = -10521$ W. Note that if the core loss resistance R_0 had not been not neglected, it would have introduced an additional loss component P_0:

$$P_0 = 3\frac{V_0^2}{R_0}\,. \tag{2.21}$$

The analysis of the equivalent circuit would have followed the same steps as above. These power flow calculations are illustrated with a Sankey diagram in Fig. 2.3. Finally, the *shaft torque* is calculated by

$$T_{sh} = \frac{P_{sh}}{\omega_{rm}} = \frac{P_{sh}}{\frac{2}{P}\omega_r} = \frac{P_{sh}}{\frac{2}{P}(1-s)\omega_e} \tag{2.22}$$

which is equal to $(-11500)(2)/(1.03)/(377) = -59.2$ N-m.

2.1.5. Calculating the efficiency. The machine's efficiency can be calculated by one of the following expressions:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = 1 - \frac{P_{\text{loss}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}}\,. \tag{2.23}$$

It is up to us to decide which one is more convenient. It is important to remember that the input and output sides depend on the mode of operation! For motor action, the input is the stator and the output is the shaft. For generator action, it is the other way around. The shaft (input) power is -11500 W. The stator (output) power has already been calculated as -10521 W. Alternatively, we could obtain this by

$$\begin{aligned} P_s &= 3\text{Re}\{\tilde{V}_s \tilde{I}_s^*\} \\ &= (3)(138.6)(29.6)\cos(148.9^\circ) = -10.5 \text{ kW} \end{aligned} \tag{2.24}$$

Therefore, the machine's efficiency is

$$\eta = \frac{10521}{11500} = 91.5\%\,.$$

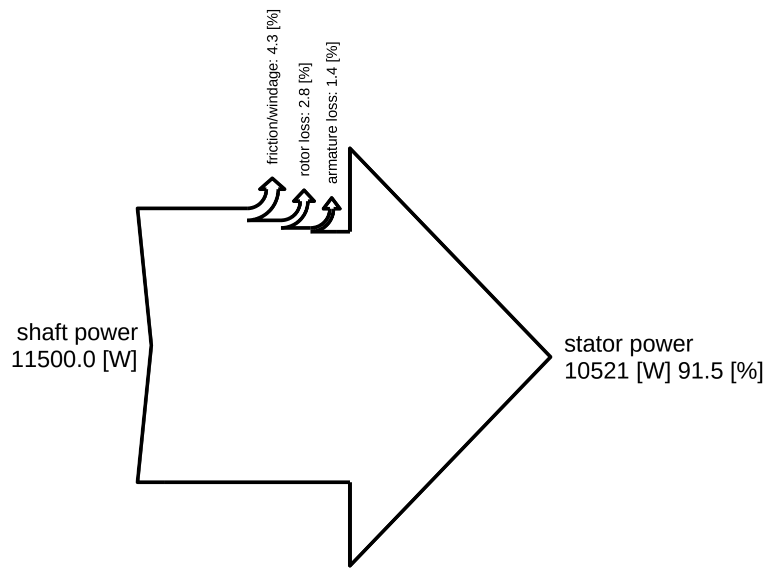

FIGURE 2.3. Sankey diagram of power flow in a squirrel-cage induction generator. (The core loss due to magnetic hysteresis and eddy currents is ignored.)

It is important to observe that *the efficiency depends on the operating point, so it varies according to how much power is being transferred.*

2.1.6. Calculating reactive power and the power factor. As electrical engineers, we are particularly interested in the reactive power consumed by the generator. As we will see, this type of generator typically draws a substantial amount of reactive power from the system. This can cause problems especially in *weak grids*, that is, transmission systems with relatively high impedance, because the reactive power consumption may lead to unacceptably low voltage levels. To improve the situation, power factor correcting capacitor banks are used, as shown in Fig. 2.1. The reactive power consumption can be calculated by

$$\begin{aligned} Q_s &= 3\mathrm{Im}\{\tilde{V}_s \tilde{I}_s^*\} \\ &= (3)(138.6)(29.6)\sin(148.9°) = 6.4 \text{ kVAr} \end{aligned} \tag{2.25}$$

We can also calculate a power factor value. It should be noted that usually we compute power factor for loads, which consume real power. For a generator, the power factor can be defined as

$$\mathrm{pf} = \frac{|P_s|}{S_s} = \frac{|P_s|}{\sqrt{P_s^2 + Q_s^2}} \,. \tag{2.26}$$

So, in this example, the power factor is $(10.5)/(\sqrt{10.5^2 + 6.4^2}) = 0.85$.

2.1.7. Calculating the torque-speed curve. Usually the induction motor operation is captured by a plot of the torque as a function of rotor speed. In the previous section we explained the process of calculating torque for a given value of slip. All we need to do to obtain the curve is to calculate the torque for a range of slip values, which correspond to a range of rotor speeds. For example, we could use Method 3 to obtain the rotor current from (2.13), and combine with (2.15) and (2.22) to obtain the following expression for torque:

$$T_e = 3\frac{P}{2}\frac{V_{\text{Th}}^2 R_2}{(s\omega_e)\left[\left(R_{\text{Th}} + \frac{R_2}{s}\right)^2 + \left(X_{\text{Th}} + X_2\right)^2\right]} \tag{2.27}$$

where we have neglected friction and windage losses. We call this the *electromagnetic torque* to distinguish it from the true shaft torque, which accounts for friction and windage losses. This is plotted in Fig. 2.4, and it usually looks like this irrespective of motor size and rating. It is extremely important to understand that a squirrel-cage machine only uses a fraction of this speed range under normal operating conditions. For motor action, the machine operates for speeds slightly less than synchronous, i.e., for $\omega_r < \omega_e$, but not very far away. This means that the slip has a small positive value. For generator action, the machine operates for speeds slightly above synchronous, i.e., for $\omega_r > \omega_e$, but not very far away. This means that the slip has a small negative value. In addition, it operates away from the peak points where maximum positive or negative torque is obtained, which are called *pull-out torque* points. For this example, the peak points are obtained for values of slip equal to ± 0.175 (it can be shown that the two slip values that give maximum/minimum torque are always of the form $\pm s_m$). Nevertheless, rated operation is obtained typically for a much smaller value of slip, on the order of 3–5%. Remember, our machine was rated for 15 HP = 11.2 kW, which is close to the power output we obtained previously for a -3% slip. The actual operating speed depends on the mechanical load on the shaft, and can be determined by intersecting the load's torque curve with the machine's torque curve. (At the equilibrium, these two torques must be exactly equal to each other.) Due to the steep slope of the torque curve around the synchronous speed, the rotor speed does not vary much regardless of mechanical load, and the squirrel-cage induction machine is essentially a *fixed-speed* machine.

2.2. Wind energy conversion with squirrel-cage induction machines

We will now see how a squirrel-cage induction generator can be used to capture the energy of the wind. We proceed with a simple example that illustrates the basic ideas.

Example 2.2. Design a wind turbine based on the previously analyzed 15-HP squirrel-cage induction generator.

What this example is asking is to determine ballpark figures for the radius of the blades and the gearbox ratio for such a (small-size) wind turbine, under some reasonable assumptions that must be made because of lack of information. What is certain is that this will be a fixed-speed turbine, whose high-speed shaft will be spinning a little higher than 1800 rpm, say, at 1855 rpm for rated power (this corresponds to -3% slip). Let's assume that: (i) the performance coefficient of the blades has a maximum value $C_{p,\max} = 0.43$ for an optimal tip-speed ratio $\lambda^* = 7.6$, and that it is expressed by a polynomial function $C_p(\lambda) = \left(0.185\lambda^4 - 5.28\lambda^3 + 40.7\lambda^2 - 28.6\lambda - 6.6\right) \times 10^{-3}$ for $3 \le \lambda \le 13$, which is plotted in Fig. 2.5; (ii) rated electrical power 11 kW is obtained when the wind speed is

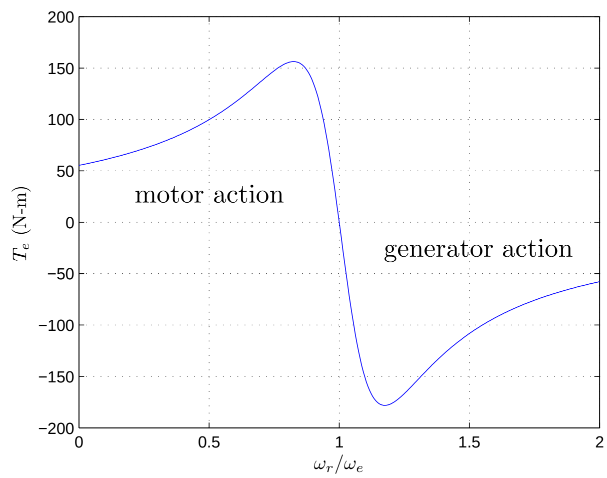

FIGURE 2.4. Torque-speed characteristic of a squirrel-cage induction machine.

10 m/s; (iii) rated power is obtained under the optimal tip-speed ratio; (iv) the gearbox efficiency is $\eta_{gb} = 90\%$; (v) the generator efficiency is $\eta_g = 91.5\%$. We use a modified version of (1.11) that reflects our assumptions,

$$P_s = \eta_{gb}\eta_g \frac{1}{2} C_{p,\max} \rho \pi R_d^2 v_1^3$$

which can be solved for the radius: $R_d = 4.0$ m. The gearbox ratio can be found from the optimal tip-speed ratio assumption, which requires that $\lambda^* = \omega_w R_d / v_1$, so: $\omega_w = (7.6)(10)/(4) = 19$ rad/s. The gearbox ratio is: $G = \omega_{rm}/\omega_w = (1.03)(188.5)/(19) = 10.2$.

It is useful to plot the electromagnetic torque-speed characteristic of this machine, and superimpose the mechanical torque from the wind, shown in Fig. 2.6. This can be achieved with a simple Matlab script, like the one appended below. From this, one can make several interesting observations: (i) the operating condition for any wind speed is found by the intersection of the corresponding red curve (mechanical torque) with the blue line (electromagnetic torque); (ii) the generator produces power from about 6 m/s to 10 m/s (the rated wind speed); (iii) the power of the wind for lower wind speeds (e.g., 4 m/s) cannot be harvested; (iv) the intersection of the 10 m/s curve with the electromagnetic torque does not correspond to the peak value of the torque, even though we assumed that this point corresponds to the peak of the $C_p(\lambda)$ curve; at this point, maximum power is extracted from the wind but this happens at a value of torque somewhat smaller than its maximum. This can be verified by plotting the power vs. speed curve, shown in Fig. 2.7. Our design is correct; the wind turbine does indeed output rated electrical power 11 kW when the wind speed is 10 m/s. Nevertheless, for wind speeds less than 10 m/s,

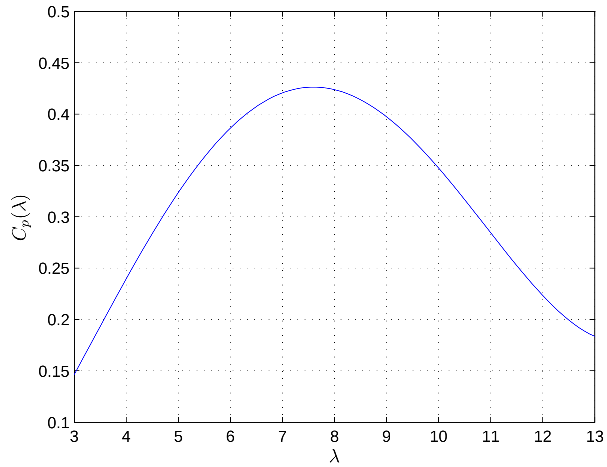

FIGURE 2.5. Blades' coefficient of performance for the turbine of Example 2.2.

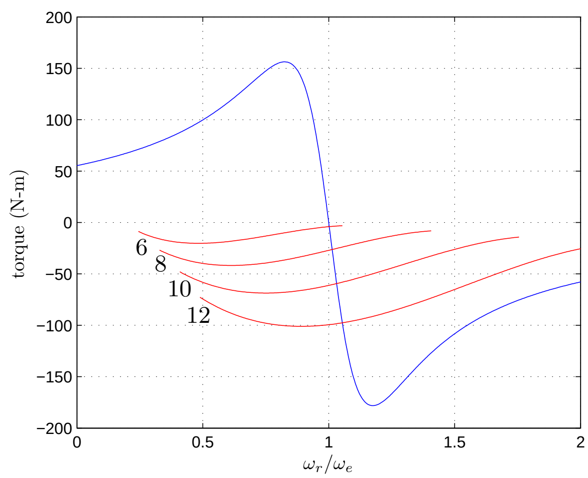

FIGURE 2.6. Electromagnetic torque-speed characteristic and mechanical torque for wind speeds $v_1 = \{6, 8, 10, 12\}$ m/s.

the power is less than the corresponding peak values. It turns out that we can do better using the topology that is described in the next chapter.

```
% MATLAB SCRIPT THAT COMPUTES TORQUE & POWER VS. SPEED

% machine parameters
we = 2*pi*60; %rad/s
R1 = 0.06;    %ohm
R2 = 0.15;
```

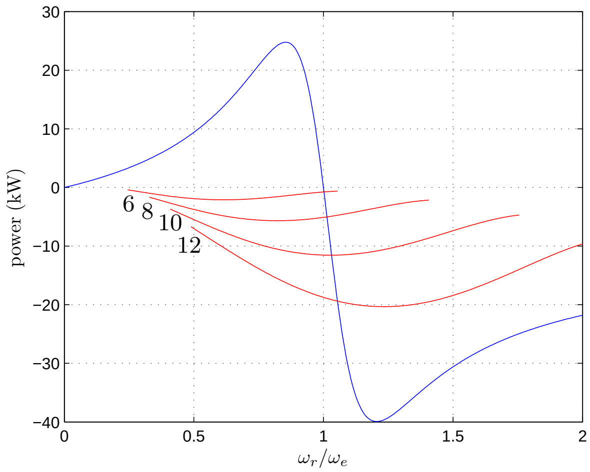

FIGURE 2.7. Electromagnetic power-speed characteristic and mechanical power for wind speeds $v_1 = \{6, 8, 10, 12\}$ m/s.

```
X1 = 0.44;
X2 = 0.43;
X0 = 12.6;
P = 4;
Vs = 138.6;   % l-n rms

Pwloss = 500;   % friction & windage loss

% gearbox parameters
G = 10.2;
eta_gb = 0.9;

% blades & air
R = 4.0;
rho = 1.25;
Ad = pi*R^2;

% wind speed
vw = 6:2:12;

% rotor speed
wr = we*linspace(0,2,500);
s = (we - wr)/we;

% Thevenin voltage & impedance
Vth = j*X0/(R1 + j*X1 + j*X0)*Vs;
Zth = 1/(1/(R1 + j*X1) + 1/(j*X0));
Rth = real(Zth);
```

```
Xth = imag(Zth);

% torque and power vs. speed characteristics
Te = 3*P/2*abs(Vth)^2*R2./(s*we)./...
    ((Rth + R2./s).^2 + (Xth + X2)^2);
Pe = Te.*wr*2/P;

figure(1), clf
% ... plot torque vs. speed commands go here ...

figure(2), clf
% ... plot power vs. speed commands go here ...

% wind power and torque
wrm = 2/P*wr;
ww = wrm/G;  % blade speed

for k=1:length(vw)
    lambda = ww*R/vw(k);
    Cp = SCIG_Cp_fun(lambda); % this function computes Cp(lambda)
    Pw = 0.5*rho*Cp*Ad*vw(k)^3; % wind power
    Psh = -Pw*eta_gb; % shaft power at the high-speed side
    Pm = Psh + Pwloss; % mechanical power entering the
             % equivalent circuit
    Tm = Pm./wrm; % mechanical torque

    % ...superimpose wind torque on figure(1) commands go here...

    % ...superimpose wind power on figure(2) commands go here...

end
```

We are also interested in the generator's terminal quantities, such as the stator current, the power factor, and the efficiency. These are plotted in Fig. 2.8. An important thing to observe is that the power factor will be quite low, especially for low and moderate wind speeds, hence necessitating the installation of a power factor correcting capacitor bank. We draw the following conclusions from the above analysis of the Type-1 topology: (i) It is essentially a fixed-speed turbine, so it does not harvest the maximum possible energy from the wind. Nevertheless, in some turbines the number of magnetic poles can be changed dynamically by reconfiguring the connection of the stator windings, thus allowing operation over a wider wind speed range. (ii) It is a simple topology that uses a low-maintenance machine (a squirrel-cage induction generator) but requires capacitor banks to improve the power factor. (iii) Because the rotor speed remains constant, variations in wind power due to fluctuating wind speed pass straight through to the electrical system and can cause undesirable effects, such as *voltage flicker*. (iv) When faults happen in the power grid (such as abrupt voltage sags due to short-circuits), the squirrel-cage machine tends to draw relatively large transient currents, which can be problematic.

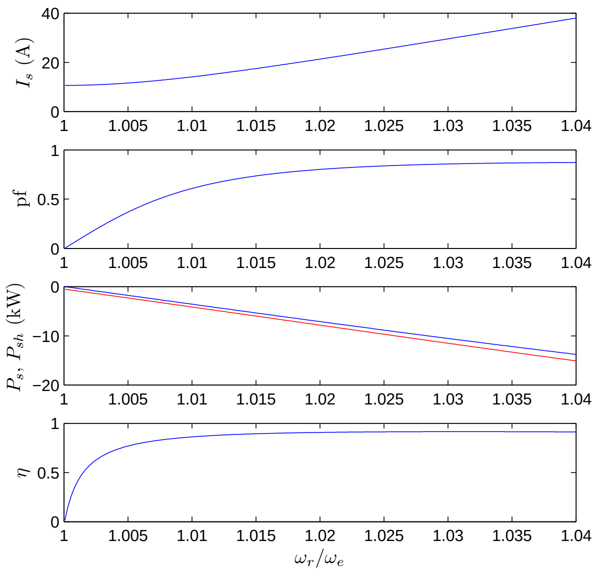

FIGURE 2.8. Plots of stator current (rms value), power factor, stator power and shaft power, and efficiency $\eta = P_s/P_{sh}$.

One must be extremely careful *not* to use the equivalent circuit for such analyses, because it is only valid for steady-state operation but rather useless for dynamic analysis of electric machinery.

2.3. Exercises

(1) A three-phase four-pole squirrel-cage induction generator is connected directly to a 60-Hz power system, and is driven by a prime mover that provides constant mechanical torque. The generator's speed is 1820 rpm. Suddenly, a voltage sag occurs; the new voltage rms value is related to the old voltage rms value by $V_2 = 0.9V_1$. The new value of speed is approximately:
   (a) 1805 rpm
   (b) 1815 rpm
   (c) 1825 rpm
   (d) 1835 rpm

(2) Modify the equivalent circuit analysis methods of Chapter 2, this time including a core loss resistor $R_0 = 10\ \Omega$. Also modify the Matlab code and re-plot Figs. 2.6 and 2.7.

CHAPTER 3

Wound-Rotor Induction Generators

This type of wind turbine, also referred to as a Type-2 turbine, uses a *wound-rotor induction generator*. This is different from the squirrel-cage induction generator because it has windings on the rotor that are not short-circuited, and which can be accessed via *slip rings* and *brushes*, as shown in Fig. 3.1. The rotor windings are then connected to an external three-phase resistor bank. The resistor bank contains variable resistors, which we can control from zero to maximum resistance R_e per phase. The control logic will be described in more detail later on. For higher reliability, we avoid using mechanically varying resistors. Instead, this control is achieved with a suitable power electronics converter; the actual resistance has a fixed value, but the electronics make it “look like” it is variable from the rotor side. A variant of this topology is one where the external resistors and power electronics are mounted on the machine’s rotor. This eliminates the slip rings and brushes, which require regular maintenance, but now control signals must be transmitted wirelessly to the rotor for adjusting the external resistance, and extra heat is dissipated inside the rotor (this makes cooling the machine more challenging).

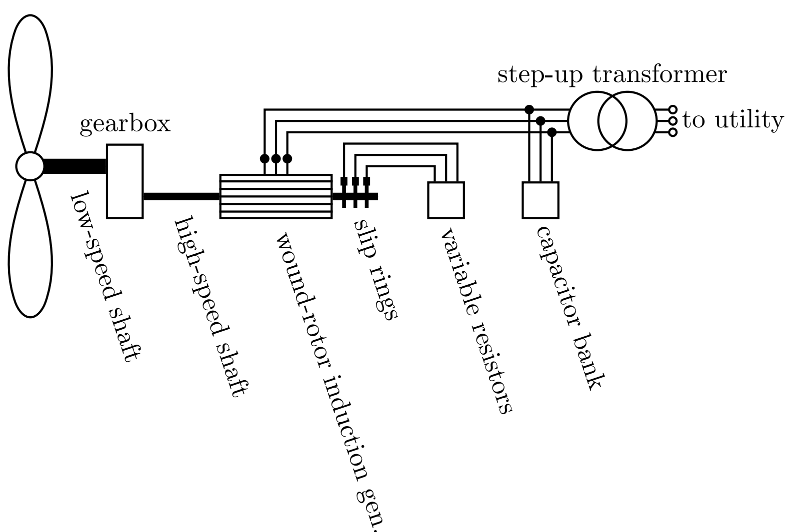

FIGURE 3.1. Topology of a Type-2 wind generator. (Note: A thyristor-based “soft-starter” power electronics converter is sometimes used, which is not shown here because it does not affect the steady-state operation. Also not shown is the parking brake.)

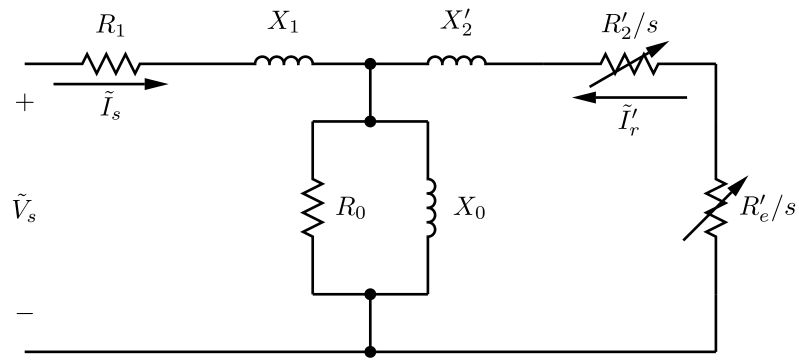

FIGURE 3.2. Equivalent steady-state circuit of the wound-rotor induction machine.

3.1. Analyzing the equivalent circuit

The equivalent circuit of this type of generator is shown in Fig. 3.2. There are two notable differences from the squirrel-cage equivalent circuit of Fig. 2.2: (i) The first difference is that there is a variable external resistance R_e'/s connected at the rotor side. Its variability stems from two sources, namely, the slip and the fact that we can control its value R_e. (ii) The second difference is that the rotor parameters are now primed, e.g., X_2' or $\tilde{I}_r'$. This is because all rotor quantities must be reflected to the stator side, in exactly the same manner as for the transformer's equivalent circuit. The transformation depends on the *turns ratio* between the stator and rotor windings. In particular, if the external resistor has an actual value of R_e Ω, then the value used in the equivalent circuit is $R_e' = (N_s/N_r)^2 R_e$. Similarly, the reflected rotor current is $\tilde{I}_r' = (N_r/N_s)\tilde{I}_r$.

This circuit can be solved using the same methods that were applied to the squirrel-cage machine. We just need to substitute R_2 by $R'_{2,\text{tot}} = R_2' + R_e'$ in the equations. In particular, the rotor circuits and the external resistance waste an amount of power equal to $3I_r'^2 R'_{2,\text{tot}}$ as heat. The other component, $3I_r'^2 \frac{1-s}{s} R'_{2,\text{tot}}$, is related to electromechanical energy conversion. The salient characteristic of a wound-rotor induction generator is that *its torque-speed curve can be modified by adjusting the value of external resistance*. This is classically illustrated with a plot such as the one in Fig. 3.3, which was obtained assuming that we have a 15-HP wound-rotor machine with the same parameters as the squirrel-cage machine in the previous chapter. The torque-speed curve is "stretched" outwards by increasing R_e. It turns out that the peak points occur for values of slip that are directly proportional to $R_2' + R_e'$, and that the corresponding pull-out torque remains constant. (Exercise 1 asks you to prove this.)

3.2. Wind energy conversion with wound-rotor machines

Nevertheless, for wind energy conversion, we are more interested in the power-speed curve, which has the interesting shape shown in Fig. 3.4. Apparently, we can use this property to operate this generator as a *variable-speed wind turbine*, which will allow us to capture more energy out of the wind stream. What we want is to match the wind's power output at any given wind speed with the appropriate

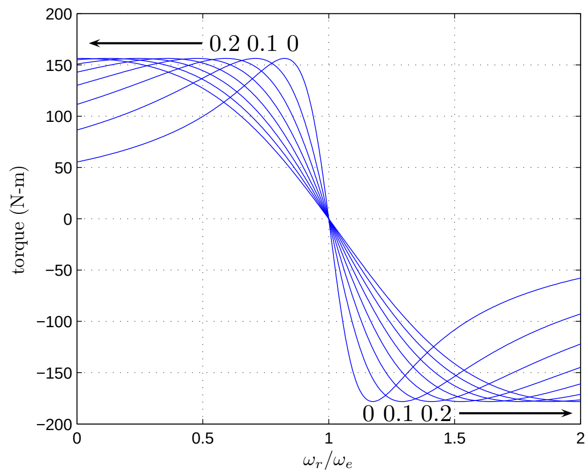

FIGURE 3.3. Torque-speed characteristic of a wound-rotor induction machine for external resistance $R'_e = \{0, 0.1, 0.2, \ldots, 0.7\}$ Ω.

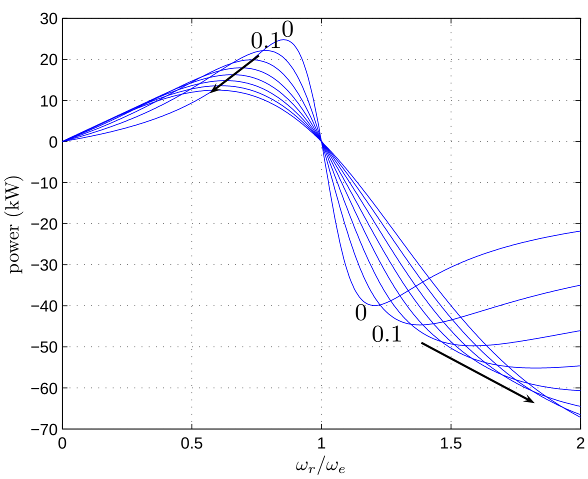

FIGURE 3.4. Power-speed characteristic of a wound-rotor induction machine for external resistance $R'_e = \{0, 0.1, 0.2, \ldots, 0.7\}$ Ω.

external resistance in order to operate at the optimum tip-speed ratio. The example that follows shows how to design a wind turbine to achieve this.

Table 3.1. Setpoints of external resistance, stator power output, and efficiency.

v_w (m/s)	R'_e (Ω)	$-P_s$ (kW)	η_g
6–8	0	1.8–5.3	0.77–0.89
8.5	0.4	6.3	0.87
9	0.75	7.4	0.83
9.5	1.05	8.2	0.79
10	1.33	9	0.76

Example 3.1. Design a wind turbine based on a 15-HP wound-rotor induction generator.

This is similar to Example 2.2, but now we will use the variable external resistance to vary the slip. Let's assume that when the wind speed is 8 m/s we operate with the optimal tip-speed ratio, and with a small slip that we can neglect in the calculation (its effect will be insignificant). For wind speeds less than 8 m/s the external resistance will be kept to zero, but as the wind speed increases further we will start increasing R_e. If the rotor speed is close to synchronous for 8 m/s, then for 10 m/s it will have to be $10/8 = 1.25$ times higher, or the slip will be -25%. Let's also maintain the same radius, $R_d = 4$ m. Other parameters, such as the performance coefficient function or the gearbox's efficiency, will be the same as before. (A more accurate calculation would use a gearbox efficiency that varies with speed, but we will assume it is constant here for simplicity.) We can first find the gearbox ratio from $\lambda^* = \omega_w R_d / v_1$, so: $\omega_w = (7.6)(8)/(4) = 15.2$ rad/s. The gearbox ratio is then: $G = \omega_{rm}/\omega_w = (188.5)/(15.2) = 12.4$. The next step is to determine the value of R'_e for each wind speed for maximum power extraction. This can be done with a simple trial-and-error process. The results are entered in Table 3.1, and are also shown in graphical form in Fig. 3.5.

Unfortunately, a critical look at the results of Example 3.1 reveals that the performance is perhaps unacceptable. The physics of induction machines are working against us. Even though we are harvesting slightly more energy from the wind than before, this does not reach the stator side because it is dissipated as heat (ohmic loss) in the rotor-side circuits (especially for the higher wind speeds). In fact, we can derive two simple formulas that can explain what is happening quite clearly. By combining (2.15), (2.17), and (2.18), it can be seen that

$$\frac{P_{ag}}{P_m} = \frac{1}{1-s} \tag{3.1}$$

and

$$\frac{P_{ag}}{P_d} = \frac{1}{s} \tag{3.2}$$

For small negative values of slip, almost all of the mechanical shaft power reaches the air gap. But as the slip starts to decrease (becomes more negative), the losses in the rotor increase. In other words, the machine's rotor is more efficient when the

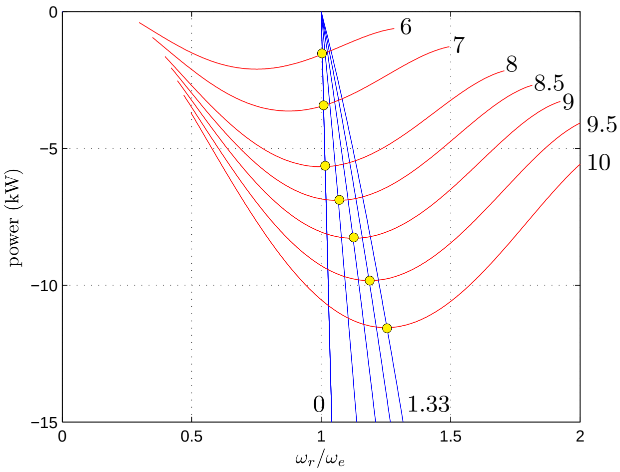

FIGURE 3.5. Power-speed characteristics of a wound-rotor induction machine (blue=electromagnetic power for various external resistances, red=mechanical power for various wind speeds), and operating points denoted by yellow circles.

rotor speed is closer to synchronous speed. In this example, we let the slip go as far as −25%, which led to a significant reduction in efficiency. A better design would be one where the slip is limited to a smaller range, say, no less than −10%, and this is what is typically done in practice; however, this kind of permissible speed range is not optimum for a wind turbine application. The type of turbine described in the next chapter can operate over a much wider speed range. Finally, it should be noted that the speed range of the Type-2 turbine can be improved by changing the number of magnetic poles (thus changing the synchronous speed), which is achieved by altering the connections of the stator windings on the fly.

3.3. Exercises

(1) For a wound-rotor induction machine, show that the slip values that correspond to maximum positive or negative torque are of the form $s_m = \pm(R_2' + R_e')C$, where $C > 0$ is a constant that does not depend on the rotor-side resistance. Also show that the maximum torque values do not depend on rotor-side resistance. [Hint: Use (2.27) to solve $\partial T_e/\partial s = 0$.]
(2) Verify that the numbers in Table 3.1, on page 26, are correct.
(3) Rework example 3.1 so that the slip does not become less than −10%, as discussed on page 27. Recompute the performance parameters shown in Table 3.1. Also compute the power output and efficiency for the same range of wind speed (compute discrete values every 1 m/s) for the Type-1 turbine of Chapter 2 and compare with the Type-2 turbine performance.

CHAPTER 4

Doubly-Fed Induction Generators

The topology of a *doubly-fed induction generator* (DFIG), also called a Type-3 turbine, is shown in Fig. 4.1. Similar to the Type-2 turbine, this topology uses a wound-rotor induction generator, but the difference lies in what is connected to the rotor. The DFIG's rotor is actually connected to the power system via an appropriate power electronics topology. The most common topology used is a *back-to-back ac-dc-ac converter*, which comprises two separate *bi-directional* converters coupled with a *dc link*. More details about the operation of the power electronics can be found in Section 12.5.4.2 of the textbook [**6**], as well as in books dedicated to the subject of power electronics [**8–10**]. The role of the *rotor-side converter*, which operates based on an advanced motor drive control, is dual: (i) it controls the frequency of the currents that flow in the rotor windings, so that synchronism is maintained between the stator and rotor magnetic fields at all times (this is achieved automatically by the laws of physics in the squirrel-cage and wound-rotor induction machine with external resistor, but now it must be done "manually"); (ii) it controls the magnitude and phase of the currents that flow in the rotor windings, and indirectly by doing this it controls the real and reactive power that the wind turbine provides to the system. In the analysis that will follow we will discuss some of the basic principles of its control strategy. We will not, however, delve into the operation of the *grid-side converter* in detail. It suffices to say that the grid-side converter has the role of transferring power to and from the rotor side, thus maintaining the dc-link capacitor at constant voltage. Both converters will be treated as ideal voltage sources, capable of generating any arbitrary voltage phasor (up to a maximum voltage limit that depends on the dc-link voltage) of arbitrary frequency (especially necessary for the rotor-side converter). The step-up transformer is a *three-winding transformer*, with two low-voltage windings for the stator and rotor, and a medium-voltage winding for connection to the wind farm collection system.

4.1. Insights from the DFIG's equivalent circuit

As usual we proceed with the equivalent circuit of this topology, shown in Fig. 4.2. This is the same as the circuit of the Type-2 topology, with one difference: the external resistor is now replaced by the rotor voltage divided by the slip, $\tilde{V}_r'/s$. Also note that this voltage is primed, i.e., reflected to the stator side, so its relationship with the actual voltage at the rotor terminals is $\tilde{V}_r' = (N_s/N_r)\tilde{V}_r$. In the equivalent circuit, all voltages and currents have the same frequency, which is the stator (power grid) frequency. In reality the rotor circuits have voltages and currents of slip frequency, as was discussed previously for the squirrel-cage machine. There is, however, a small problem with this circuit that should be pointed out: it is valid for all nonzero slip values but breaks down for $s = 0$ due to divisions

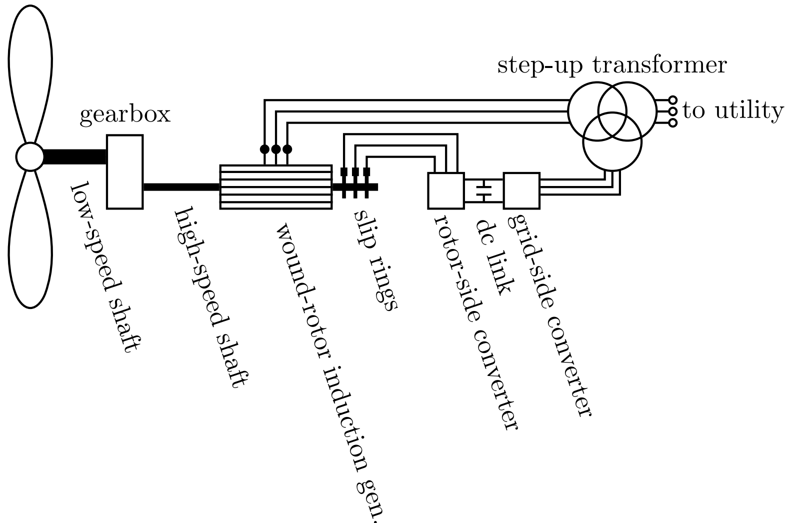

FIGURE 4.1. Topology of a Type-3 wind generator. (Note: Not shown is an auxiliary *crowbar* circuit that is used to protect the power electronics during power system faults.)

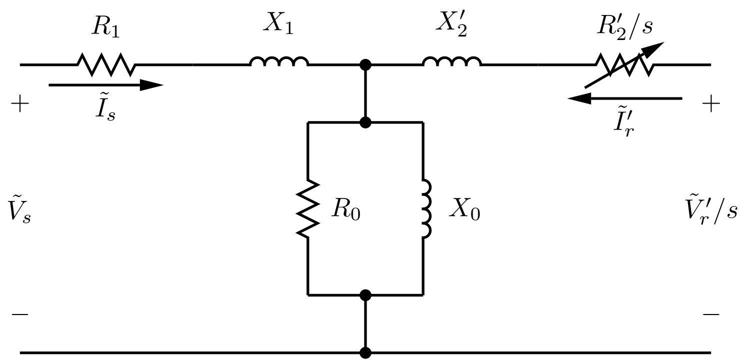

FIGURE 4.2. Equivalent steady-state circuit of the doubly-fed induction generator.

by zero. This is a real issue for the DFIG because—as we will see in the ensuing analysis—it operates for a range of slips from negative to positive values (so zero is included). For $s = 0$, the rotor-side converter generates a dc voltage, and the rotor circuits have dc currents. Let us not worry about this for the moment; we will deal with this issue later on.

One of the main reasons why the DFIG has become a very common topology for wind energy conversion becomes obvious once the flows of power are examined. We will ignore R_0 in the analysis. First, we write expressions for the stator and rotor voltages by looking at the equivalent circuit and applying Kirchhoff's voltage law twice:

$$\tilde{V}_s = R_1\tilde{I}_s + jX_1\tilde{I}_s + jX_0\tilde{I}_m \tag{4.1}$$

$$\tilde{V}_r' = R_2'\tilde{I}_r' + jsX_2'\tilde{I}_r' + jsX_0\tilde{I}_m \tag{4.2}$$

where $\tilde{I}_m = \tilde{I}_s + \tilde{I}_r'$. Note that the rotor-side voltage equation was obtained by multiplying both sides of the KVL equation by s, so that it no longer is in the denominators. We are interested in the real powers that flow into the stator and rotor side, P_s and P_r, respectively. These are

$$P_s = 3\text{Re}\{\tilde{V}_s\tilde{I}_s^*\} \tag{4.3}$$

$$P_r = 3\text{Re}\{\tilde{V}_r'\tilde{I}_r'^*\} \tag{4.4}$$

so by using the voltage equations above:

$$P_s = 3R_1I_s^2 + 3\text{Re}\{jX_0\tilde{I}_m\tilde{I}_s^*\} \tag{4.5}$$

$$P_r = 3R_2'I_r'^2 + 3\text{Re}\{jsX_0\tilde{I}_m\tilde{I}_r'^*\} \tag{4.6}$$

or

$$P_s = 3R_1I_s^2 + 3\text{Re}\{jX_0(\tilde{I}_s + \tilde{I}_r')\tilde{I}_s^*\} \tag{4.7}$$

$$P_r = 3R_2'I_r'^2 + 3\text{Re}\{jsX_0(\tilde{I}_s + \tilde{I}_r')\tilde{I}_r'^*\} \tag{4.8}$$

and finally

$$P_s = 3R_1I_s^2 + 3\text{Re}\{jX_0\tilde{I}_r'\tilde{I}_s^*\} \tag{4.9}$$

$$P_r = 3R_2'I_r'^2 + 3\text{Re}\{jsX_0\tilde{I}_s\tilde{I}_r'^*\} \tag{4.10}$$

The quantity $P_s - 3R_1I_s^2$ is what flows to the rotor via the air gap; previously we named this the *air gap power* P_{ag}. The quantity $P_r - 3R_2'I_r'^2$ is called the *slip power* P_{sl}. Using some elementary complex algebra manipulations, it can be shown that

$$P_{sl} = -sP_{ag} \tag{4.11}$$

and if we ignore the relatively small resistive losses we can also approximately say that

$$P_r \approx -sP_s \tag{4.12}$$

Obviously, the mechanical power must be (from conservation of energy considerations)

$$P_m = P_{ag} + P_{sl} = (1-s)P_{ag} \tag{4.13}$$

From this analysis we can draw the following conclusions:

- When $s > 0$ (this is called *sub-synchronous* operation) the rotor side power has opposite sign than the stator side power. So, for generator action ($P_s < 0$) the rotor is absorbing power.
- When $s < 0$ (this is called *super-synchronous* operation) the rotor side power has the same sign as the stator side power. So, for generator action ($P_s < 0$) the rotor is generating power.
- The slip power increases with $|s|$. In the Type-2 turbine, this power was lost as heat in the external resistance. In the Type-3 turbine, we can *recover* this power using the power electronics, and feed it back to the system. This permits efficient operation for much higher values of slip than before.
- The power coming from the shaft (for generator action) splits between the stator and rotor sides. For small values of $|s|$, the stator side carries the bulk of the power. Therefore, *the power electronics that are connected to the rotor side need to be rated for only the slip power!* The use of a

partially rated converter leads to significant cost savings, and simplifies the design of the power electronics. It is in fact one of the main advantages that have led to the prevalence of the Type-3 wind turbine. Typically, commercially available DFIG turbines operate within a ±30% slip range.

- Something very interesting takes place for sub-synchronous operation. The situation is easier to understand with a numerical example. Let's assume that the shaft power is '-1' (generator action, units omitted), and that the slip is 20%. Then the air gap power will be $(-1)/(1-0.2) = -1.25$, which implies that the stator outputs more power than is generated from the mechanical side! This extra power comes from the rotor side, which absorbs it from the system. In other words, an amount of power equal to '0.25' circulates within the topology.
- For super-synchronous operation, the situation is as follows. Let's assume that the shaft power is '-1' (generator action, units omitted), and that the slip is -20%. Then the air gap power will be $(-1)/(1+0.2) = -0.83$ and the rotor power is -0.17. In this case, the shaft power splits in two parts, with the bulk of the power flowing through the stator side. The two components of power meet again at the transformer, where they are combined and supplied to the power system.

The special case of synchronous speed operation ($s = 0$) appears to be problematic, due to a division by zero in the equivalent circuit. However, when we wrote the rotor voltage equation in the form (4.2), we indirectly solved this issue. In this special case, the rotor voltage equation becomes

$$\tilde{V}_r' = R_2' \tilde{I}_r' \tag{4.14}$$

So, at this operating point, the rotor-side voltage needs to be just enough to drive the current through the rotor winding resistance, and the rotor-side converter supplies only a small amount of power equal to the ohmic loss on the rotor-side resistance. It is important to understand that, even if the above equation uses phasors that represent ac quantities, in reality the rotor quantities are dc when $s = 0$. For example, the rotor's line-to-neutral voltages and currents are

$$v_{ar}' = \sqrt{2} V_r' \cos(\theta) = R_2' i_{ar}' \tag{4.15}$$

$$v_{br}' = \sqrt{2} V_r' \cos(\theta - \tfrac{2\pi}{3}) = R_2' i_{br}' \tag{4.16}$$

$$v_{cr}' = \sqrt{2} V_r' \cos(\theta + \tfrac{2\pi}{3}) = R_2' i_{cr}' \tag{4.17}$$

where θ is a constant angle. These equations show the relationship between the dc values and the rms values (V_r', I_r') of the fictitious ac quantities that are used in the equivalent circuit.

4.2. Wind energy conversion with DFIGs

A typical power vs. wind speed characteristic of a Type-3 wind turbine is shown in Fig. 4.3 [**1**, **4**], which is a modified version of Fig. 1.4. The following operating modes are present:

- From point B to C, the turbine's rotational speed is variable, and the control system tries to maintain the optimum tip-speed ratio by keeping the blades' ω_w proportional to v_1. The power production follows a quasi-cubic trajectory due to the losses in the drivetrain. The generator will

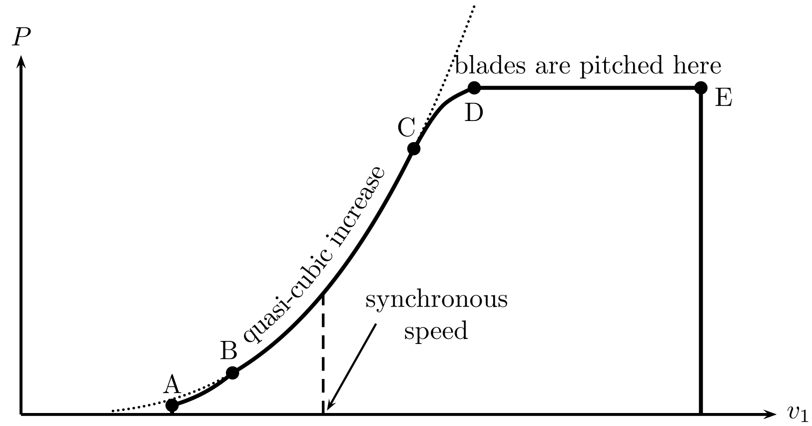

FIGURE 4.3. Type-3 turbine power output as a function of wind speed.

rotate at synchronous speed for a wind speed somewhere in between the two limits, and in sub-synchronous or super-synchronous mode otherwise.

- From point A (cut-in speed) to B, the turbine operates as a fixed-speed machine, at minimum speed $\omega_{w,\min}$. Therefore, the tip-speed ratio in this range is suboptimal, and the power production deviates from its ideal cubic trajectory (represented by the dotted curve). The same thing happens from point C to D (rated speed), where the speed is kept constant at its maximum value $\omega_{w,\max}$. One reason why these two limits appear is because the rotor-side converter might not have sufficient voltage capability to drive the currents for larger speed deviations, even for the low-speed, low-power condition. (The maximum voltage that a converter can generate is related to the voltage level of the dc link. This is a design parameter and it is limited by what the dc-link capacitors can safely handle. Note that the dc-link voltage should be high enough to drive the currents required for both the rotor-side and the grid-side converter. If the three-winding transformer has unequal turns ratios, the grid-side converter voltage can be lower than the stator winding voltage.) A simple justification of this can be obtained from the equivalent circuit, by neglecting stator and rotor resistance and leakage inductances, which leads to $\tilde{V}_r' \approx s\tilde{V}_s$ or $V_r' \approx |s|V_s$. Bigger deviations from synchronous speed correspond to proportionally bigger absolute values of slip and rotor voltage.
- At point D, the generator's rated power is reached, so from point D to E (cut-out speed) the blades are pitched to maintain constant power output.

It should be noted that, in reality, the power output does not follow such a nice and smooth trajectory as shown in Fig. 4.3. Due to the constant fluctuations of wind speed, the power output looks like a cloud around the ideal curve.

4.3. Controlling a DFIG

Now let us take a look at how the DFIG is controlled. To this end, we will consider the rotor-side converter acting as a controllable current source, which can inject the appropriate currents in the rotor windings, $\tilde{I}_r'$. We can control the current's magnitude (up to a maximum value) and angle, which is measured relative to the stator voltage. Without loss of generality, we assume that the stator voltage phasor is the angular reference, $\tilde{V}_s = V_s\angle 0$, and that the rotor current phasor is $\tilde{I}'_r = I'_r\angle\theta_{ir} = I'_{ra} + jI'_{rb}$. The stator voltage magnitude V_s is imposed from the larger power system that the turbine is connected to, and for this analysis it will be assumed constant. We will also ignore the effect of R_0. Let's re-write the stator voltage equation (4.1) as follows:

$$\tilde{V}_s = (R_1 + jX_s)\tilde{I}_s + jX_0\tilde{I}'_r \tag{4.18}$$

where we defined $X_s = X_0 + X_1$. This can be solved for the stator current

$$\tilde{I}_s = \frac{\tilde{V}_s - jX_0\tilde{I}'_r}{R_1 + jX_s} \tag{4.19}$$

Typically, R_1 is much smaller than X_s, so an excellent approximation is

$$\tilde{I}_s \approx \frac{\tilde{V}_s - jX_0\tilde{I}'_r}{jX_s} \tag{4.20}$$

The complex power consumed by the generator' stator is $S_s = 3\tilde{V}_s\tilde{I}_s^*$, which becomes

$$S_s \approx 3V_s\frac{V_s + jX_0(I'_{ra} - jI'_{rb})}{-jX_s} \tag{4.21}$$

$$= j3V_s\frac{(V_s + X_0I'_{rb}) + jX_0I'_{ra}}{X_s} \tag{4.22}$$

The stator real and reactive power are then

$$P_s \approx -3\frac{X_0}{X_s}V_sI'_{ra} \tag{4.23}$$

$$Q_s \approx 3V_s\frac{V_s + X_0I'_{rb}}{X_s} \tag{4.24}$$

From these equations we can see that by controlling the rotor current phasor we can actually control the power output of the turbine! In fact, we can control independently the stator's real and reactive power by adjusting the two components of the rotor current, I'_{ra} and I'_{rb}. For generator action, I'_{ra} must be always positive, and its exact value depends on the wind speed (i.e., on how much power must be absorbed from the wind). We can also make the stator absorb or generate reactive power; if $I'_{rb} > -V_s/X_0$ the stator absorbs reactive power, and it generates reactive power otherwise. This capability is very important, because the DFIG can operate with (controllable) power factor close to unity, so it is not necessary to install power factor correcting capacitor banks, which were necessary for Type-1 and Type-2 turbines. Some turbines also provide the capability to power system operators to remotely set the desirable reactive power consumption or generation level, according to the needs of the power system (for example, in order to regulate the voltage level in the transmission system, similarly to what is done with the excitation systems of traditional synchronous generators). In practice, the positioning of the rotor current phasor at the appropriate angle θ_{ir} requires measurement of the stator voltage angle using a *phase-locked loop* and measurement of the rotor angle using a *position encoder.*

EXAMPLE 4.1. Consider a 1.8-MW DFIG wind turbine, whose stator is rated for 690 V (line-to-line), 60 Hz. The parameters of the generator are $X_1 = X_2' = 0.04\ \Omega$, $X_0 = 0.9\ \Omega$, $R_1 = R_2' = 2\ \text{m}\Omega$, and it has $P = 6$ poles. The machine's total hysteresis, eddy current, friction, and windage loss is 4 kW. Analyze the following two operating conditions, assuming rated stator voltage: (i) $P_{\text{out}} = 0.6$ MW, $Q_s = 0$, for $s = 0.19$; (ii) $P_{\text{out}} = 1.4$ MW, $Q_s = 0.1$ MVAr, for $s = -0.19$.

(i) The first operating point corresponds to sub-synchronous operation. For a 60-Hz, 6-pole machine, synchronous speed is 1200 rpm. So the generator's speed is $(1-0.19)(1200) = 972$ rpm. Using the approximation $P_r \approx -sP_s$, we obtain $P_{\text{out}} = -(P_s + P_r) \approx -(1-s)P_s$. So $P_s \approx -(0.6)/(1-0.19) = -0.74$ MW (negative sign because the stator is generating power) and $P_r \approx 0.14$ MW (positive sign because the rotor is absorbing power). An amount of power equal to 140 kW is circulating within the generator and the power electronics.

The rotor current can be computed using (4.23) and (4.24):

$$I'_{ra} \approx -\frac{P_s X_s}{3X_0 V_s} = \frac{(740000)(0.94)}{(3)(0.9)(690/\sqrt{3})} = 646.7 \text{ A}$$

and

$$I'_{rb} \approx \frac{Q_s X_s}{3X_0 V_s} - \frac{V_s}{X_0} = -\frac{690/\sqrt{3}}{0.9} = -442.6 \text{ A}$$

So, the rotor current phasor is $\tilde{I}'_r = 646.7 - j442.6 = 783.7\angle -34.4°$ A. It is interesting to note the relatively large value of current (442.6 A) that is required to maintain unity power factor at the stator side. This current component compensates the reactive power loss inside the generator. The stator current can be computed from (4.20):

$$\tilde{I}_s \approx \frac{(690/\sqrt{3}) - j(0.9)(646.7 - j442.6)}{j0.94} = 619.2\angle 180° \text{ A}$$

Since $\tilde{V}_s = (690/\sqrt{3})\angle 0$, this yields a real power production from the stator $P_s = -740$ kW. The rotor voltage can be found using (4.2):

$$\tilde{V}'_r = (0.002 + j(0.19)(0.94))(646.7 - j442.6) + \\ + j(0.19)(0.9)(-619.2) = 80.8\angle 6.2° \text{ V}$$

The real power absorbed by the rotor is

$$P_r = 3V'_r I'_r \cos(\theta_{vr} - \theta_{ir}) \\ = (3)(80.8)(783.7)(\cos 40.6°) = 144.2 \text{ kW}$$

which is close to the value estimated above (140 kW). This is supplied by the rotor-side converter. The grid-side converter will have to absorb this same amount plus an additional component to compensate for internal losses in the power electronics. For example, if we assume that each converter is 96% efficient, the amount drawn by the grid-side converter is $(144.2)/(0.96^2) = 156.5$ kW. The total real power provided to the system by the wind turbine is then $(740) - (156.5) = 583.5$ kW, which turns out to be slightly less than the original estimate of 600 kW. This is because we are now accounting for the various losses in the system with more accuracy. If the wind turbine really "needs" to provide 600 kW to the system, this can be readily achieved by its control system that can quickly adjust the rotor current I'_{ra} upwards to achieve this goal. The ohmic loss on the rotor is

$$3(I'_r)^2 R'_2 = (3)(783.7^2)(0.002) = 3.7 \text{ kW}$$

The ohmic loss on the stator is

$$3(I_s)^2 R_1 = (3)(619.2^2)(0.002) = 2.3 \text{ kW}$$

Therefore, the slip power is $(144.2) - (3.7) = 140.5$ kW, the airgap power is $(-740) - (2.3) = -742.3$ kW, and the mechanical power is $(-742.3) + (140.5) = -601.8$ kW. If we add the hysteresis, eddy current, friction, and windage loss, we obtain the total input power to the generator at its shaft, 605.8 kW. The combined efficiency of the generator and power electronics is $(583.5)/(605.8) = 96.3\%$.

The reactive power absorbed by the rotor is

$$\begin{aligned} Q_r &= 3V'_r I'_r \sin(\theta_{vr} - \theta_{ir}) \\ &= (3)(80.8)(783.7)(\sin 40.6^\circ) = 123.6 \text{ kVAr} \end{aligned}$$

This is supplied by the rotor-side converter. Note, however, that the grid-side converter does *not* have to absorb the same amount of reactive power from the grid (unlike the real power, there is no physical law that warrants the conservation of reactive power flowing through two back-to-back converters). In fact, the reactive powers of the rotor-side and grid-side converters are completely independent from each other. The grid-side converter can be commanded to absorb (or generate) its own reactive power, Q_g. The total reactive power absorbed by the combination of generator and power electronics is $Q_s + Q_g$. Of course, the total reactive power absorbed from the power system is somewhat greater than this due to the presence of the step-up transformer, which consumes some amount of reactive power as well. If we knew the impedance of this transformer, we would be able to calculate this. The advantage of using the rotor-side converter instead of the grid-side converter for reactive power control is that it acts as a reactive power "amplifier!" The rotor-side reactive power is essentially amplified by a factor close to $1/s$. To see this, you may compare the reactive power that is provided (123.6 kVAr) with the amount that appears at the stator side ($3V_s(X_0/X_s)I'_{rb} = -506.4$ kVAr), exactly balancing the $3V_s^2/X_s$ consumption amount in this case.

(ii) The second operating point corresponds to super-synchronous operation. The generator's speed is $(1 + 0.19)(1200) = 1428$ rpm. Now the powers are divided as follows: $P_s \approx (-1.4)/(1 + 0.19) = -1.18$ MW and $P_r \approx -0.22$ MW (negative sign because the rotor is also providing power).

The rotor current can be computed using (4.23) and (4.24):

$$I'_{ra} \approx -\frac{P_s X_s}{3X_0 V_s} = \frac{(1180000)(0.94)}{(3)(0.9)(690/\sqrt{3})} = 1031.2 \text{ A}$$

and

$$I'_{rb} \approx \frac{Q_s X_s}{3X_0 V_s} - \frac{V_s}{X_0} = \frac{(100000)(0.94)}{(3)(0.9)(690/\sqrt{3})} - \frac{690/\sqrt{3}}{0.9} = -355.2 \text{ A}$$

So, the rotor current phasor is $\tilde{I}'_r = 1031.2 - j355.2 = 1090.7\angle - 19.0^\circ$ A. The stator current can be computed from (4.20):

$$\tilde{I}_s \approx \frac{(690/\sqrt{3}) - j(0.9)(1031.2 - j355.2)}{j0.94} = 990.9\angle - 175.2^\circ \text{ A}$$

Since $\tilde{V}_s = (690/\sqrt{3})\angle 0$, this yields a real power production from the stator $P_s = -1.18$ MW. The rotor voltage can be found using (4.2):

$$\begin{aligned}\tilde{V}'_r = (0.002 + j(-0.19)(0.94))(1031.2 - j355.2) + \\ + j(-0.19)(0.9)(990.9\angle - 175.2^\circ) = 77.2\angle - 168.0^\circ \text{ V}\end{aligned}$$

Due to the negative slip, the rotor voltage phasor has dramatically changed direction from the previous case (almost a 180° rotation).

The real power generated by the rotor is

$$\begin{aligned}P_r &= 3V'_r I'_r \cos(\theta_{vr} - \theta_{ir}) \\ &= (3)(77.2)(1090.7)(\cos(-149^\circ)) = -216.5 \text{ kW}\end{aligned}$$

which is close to the value estimated above (220 kW). If we assume that each converter is 96% efficient, the power exiting the grid-side converter is $(216.5)(0.96^2) = 199.5$ kW. The total real power provided to the system by the wind turbine is then $(1.18) + (0.1995) \approx 1.38$ MW, which is slightly less than the original estimate of 1.4 MW. The ohmic loss on the rotor is

$$3(I'_r)^2 R'_2 = (3)(1090.7^2)(0.002) = 7.1 \text{ kW}$$

The ohmic loss on the stator is

$$3(I_s)^2 R_1 = (3)(990.9^2)(0.002) = 5.9 \text{ kW}$$

Therefore, the slip power is $(-216.5) - (7.1) = -223.6$ kW, the airgap power is $(-1180) - (5.9) = -1185.9$ kW, and the mechanical power is $(-1185.9) + (-223.6) = -1409.5$ kW. If we add the hysteresis, eddy current, friction, and windage loss, we obtain the total input power to the generator, 1413.5 kW. The combined efficiency of the generator and power electronics is $(1380)/(1413.5) = 97.6\%$.

The reactive power absorbed by the rotor is

$$\begin{aligned}Q_r &= -3V'_r I'_r \sin(\theta_{vr} - \theta_{ir}) \\ &= -(3)(77.2)(1090.7)(\sin(-149^\circ)) = 130.1 \text{ kVAr}\end{aligned}$$

It is important to understand that when the slip is negative (during super-synchronous operation), the electrical frequency of the rotor currents also becomes negative, because it is equal to $s\omega_e$. A negative frequency can be interpreted as a reversal of the direction of rotation of the rotor phasors, which now are spinning clockwise (remember, phasors are normally rotating counter-clockwise). So, in this example, the rotor current is still *lagging* the voltage by 149°, and the rotor is consuming reactive power. This explains the minus sign that appeared in the reactive power equation above.

TABLE 4.1. Wind speed statistical data and turbine power production data for Problem 1.

v_w (m/s)	probability (%)	power from turbine (kW)
0–4	—	0
4–5	8	100
5–6	9	200
6–7	9.5	320
7–8	9.5	550
8–9	9	810
9–10	8.5	1150
10–11	7.5	1410
11–12	6.5	1660
12–13	5	1770
13–25	12	1800
> 25	—	0

We conclude this chapter by summarizing the salient points of Type-3 (DFIG) wind turbines' operational characteristics: (i) They are variable-speed turbines, and so can harvest more of the available wind energy. (ii) The ability to operate at reduced rotor speeds also improves aerodynamic noise emissions. (iii) They only need a partially rated power electronics converter. (iv) Due to the variable-speed operation, the combined rotating inertia of the blades, hub, gearbox, shafts, and generator rotor acts as a kinetic energy storage buffer; so the fast variations of the wind speed are filtered and do not pass directly through to the power system, thus reducing voltage flicker and other related interconnection problems. (v) The power electronics provide the capability to control the turbine's reactive power output as well. (vi) Unlike the Type-1 and Type-2 turbines, the DFIGs can remain connected to the power system during faults, and in some cases will support the power system recovery. This capability is called *low-voltage ride-through*, and is nowadays mandated by grid interconnection regulations worldwide.

4.4. Exercises

(1) At a candidate site that is under consideration for a possible wind power plant development, the statistics of wind speed at hub height are provided in Table 4.1. The plant will contain 1.8-MW Type-3 turbines; their average power production for each wind speed range is also provided in the table (this is obtained from the turbine's data sheet). Estimate the annual energy yield of each turbine (in MWh) and the capacity factor.

(2) Consider a DFIG wind turbine, whose stator is rated for 690 V (line-to-line), 60 Hz. The parameters of the generator are $X_1 = X_2' = 0.05\ \Omega$, $X_0 = 1\ \Omega$, $R_1 = R_2' = 2\ \text{m}\Omega$, and it has $P = 6$ poles. The generator is rotating at synchronous speed, producing 1 MW, under rated voltage conditions. The machine's total hysteresis, eddy current, and windage loss is 5 kW.
   (a) Which answer best represents the split of real power between stator and rotor?
      (A) $(P_s, P_r) = (-0.5, -0.5)$ MW

(B) $(P_s, P_r) = (-0.75, -0.25)$ MW
(C) $(P_s, P_r) = (-1.0, 0.0)$ MW
(D) $(P_s, P_r) = (-1.25, 0.25)$ MW

(b) If the stator is providing 100 kVAr to the power system, compute the rotor current, $\tilde{I}_r'$.
(c) Compute the rotor voltage, $\tilde{V}_r'$.
(d) Compute the generator's shaft speed in rpm.
(e) The grid-side converter is providing 50 kVAr to the power system. How much reactive power is flowing from the rotor to the rotor-side converter?
(f) What is the power at the generator's shaft?
(g) The generator is connected to the blades through a gearbox with a gear ratio of 80:1. Assuming the gearbox is lossless, what is the torque at the low-speed shaft?
(h) What is the generator's efficiency?

(3) Consider the DFIG used in Example 4.1. We would like to analyze its operation for the point-B to point-C range shown in Fig. 4.3. At point B, the slip is 25% and the power produced by the turbine is 400 kW. At point C, the slip is −20% and the power produced is 1.5 MW. At synchronous speed (somewhere in between points B and C), the power output is 800 kW. Assume a quasi-cubic increase of power output in this speed range of the form $P(x) = a_1 x + a_2 x^2 + a_3 x^3$, where x is the *per-unit rotor speed*, ω_r/ω_e, and $P(x)$ is measured in MW.
(a) Determine the coefficients a_1, a_2, a_3 of the power output function.
(b) Write a Matlab script that calculates the same quantities that were found when solving Example 4.1, such as rotor current, rotor voltage, rotor power, losses, etc., for the entire speed range between the two points B and C. Also plot the variation of these quantities as a function of x, i.e., rotor speed. Repeat the calculations and plots three times: first for unity power factor, then for power factor 0.95 leading (define "leading" to mean that the turbine is absorbing Q, as if the load that it is connected to has a leading power factor) and 0.95 lagging (define "lagging" to mean that the turbine is generating Q). Assume that the grid-side converter is not providing any reactive power support to the system.

CHAPTER 5

Permanent-Magnet Synchronous Generators

Type-4 wind turbines are based on permanent-magnet synchronous generators (PMSGs), and are connected using the topology shown in Fig. 5.1. A *fully rated* power electronics converter that can handle the full power output of the generator is needed to interface the generator with the power system. The converter enables the decoupling of the rotational speed of the machine from the constant electrical frequency of the grid. For variable-speed operation, the stator-side converter generates ac voltages and currents of the appropriate frequency to match the rotor's speed. (Remember, in synchronous machines, the rotor must be synchronized with the magnetic field of the stator.) It can also regulate the magnitude and phase of the stator current, in order to control the electromagnetic torque of the generator, and therefore to make the rotor speed approach the value required to obtain optimal tip-speed ratio. Usually, the power electronics topology is like the one shown in Fig. 5.1, i.e., employing two back-to-back IGBT-based converters with a dc link, although other topologies can be used as well. In another quite common topology, an uncontrolled diode rectifier is connected at the stator side instead of a fully controllable converter. This helps reduce the complexity and cost of the power electronics, but does not provide as much flexibility to control the stator currents. In these notes, we will analyze the former topology, thus implicitly assuming that we can precisely adjust the stator current magnitude and phase angle.

5.1. The PMSG equivalent circuit

A very good introduction to permanent magnet synchronous machines is provided in Chapter 15 of the textbook [**6**]. It turns out that the steady-state model of a PMSG—similarly to the model of a classical synchronous generator—can be expressed simply as an internal voltage source in series with an impedance. The model's basic equation relates the terminal voltage phasor $\tilde{V}_s$ to the stator current phasor $\tilde{I}_s$ and an internal voltage phasor $\tilde{E}_{pm}$:

$$\tilde{V}_s = \tilde{E}_{pm} - (R_s + j\omega_r L_q)\tilde{I}_s \tag{5.1}$$

$\tilde{E}_{pm}$ is produced by the rotation of the permanent magnets but also depends on a component of stator current called the d-axis current and denoted by I_{ds}; it is given by

$$\tilde{E}_{pm} = \tfrac{1}{\sqrt{2}}\omega_r \left[\lambda_{pm} - (L_d - L_q)I_{ds}\right] e^{j\delta} \tag{5.2}$$

The current is assumed to have a positive direction flowing out of the generator's terminals. (Note that this is a slight difference from the convention used in the previous chapters, where stator current was assumed to be flowing into the terminals. This is just a different way of defining things, and leads to the real power being positive for generation action.) The PMSG equivalent circuit is shown in Fig. 5.2.

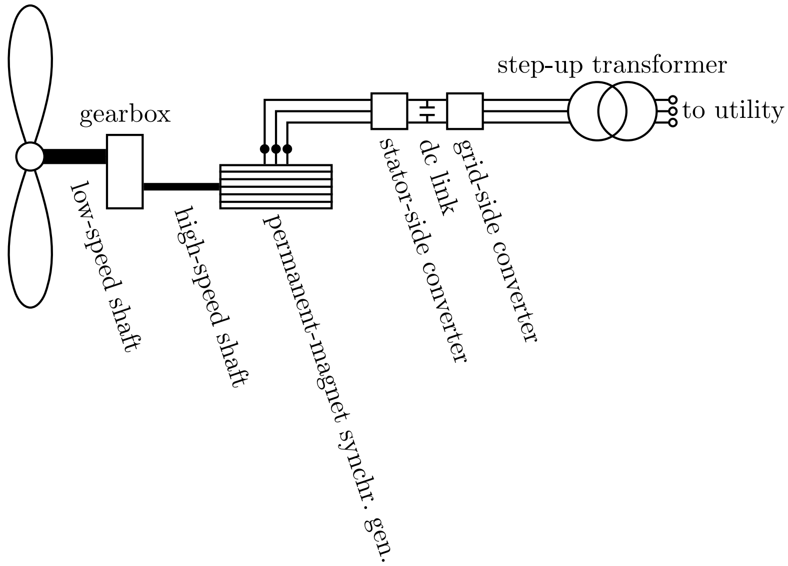

FIGURE 5.1. Topology of a Type-4 wind generator. (Note: *Direct-drive* designs do not have a gearbox; the low-speed shaft is directly connected to a PMSG with large number of poles.)

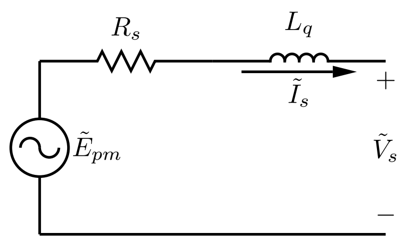

FIGURE 5.2. Equivalent steady-state circuit of a PMSG.

The above equations involve the machine's parameters and operating condition: R_s is the stator winding resistance per phase in Ω or mΩ (assuming Y-connection); L_q and L_d are the qd-axes inductances in H or mH; ω_r is the electrical rotor speed in rad/s; λ_{pm} is the constant flux linkage generated by the permanent magnets in Wb or V-s; δ measures the rotor's angle relative to the phase voltage in rad (in steady state, the rotor and the voltage have the same frequency, so δ is constant); finally, the stator current satisfies the following equation, via which the qd-axes currents are indirectly defined:

$$\sqrt{2}\tilde{I}_s e^{-j\delta} = I_{qs} - jI_{ds} \tag{5.3}$$

The interested reader can refer to [**7**] for more details about these equations and their derivation. The equations suggest the phasor diagram shown in Fig. 5.3, where the stator voltage is chosen as the angle reference. The phasor diagram contains two sets of orthogonal axes: (i) The real and imaginary axes are stationary, and they represent the coordinate system for phasors. (ii) The q- and d-axes are positioned on the rotor, and in particular the d-axis is aligned with the magnetic flux generated

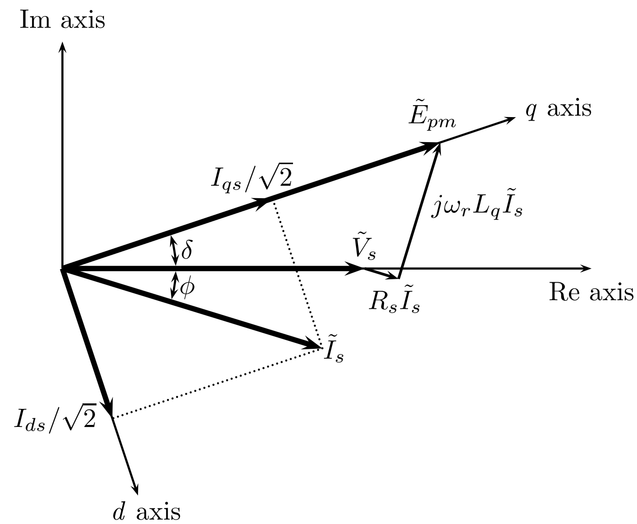

FIGURE 5.3. Phasor diagram of a PMSG. (The current angle ϕ is negative when the current is in the position shown.)

by the permanent magnets. The figure shows how the stator current phasor can be decomposed into two other components I_{qs} and I_{ds} (instead of $\mathrm{Re}\{\tilde{I}_s\}$ and $\mathrm{Im}\{\tilde{I}_s\}$) along these axes. The phasor diagram (by definition) can be interpreted as a snapshot of the situation at $t = 0$; for synchronous generators we usually define $t = 0$ as the moment when the a-phase stator voltage phasor ($\tilde{V}_s$) is horizontal. If we unfreeze time, everything in the picture will rotate counterclockwise with velocity ω_r, including the qd-axes, with the exception of the stationary real and imaginary axes.

Before we proceed, we make two interesting observations about the internal voltage phasor $\tilde{E}_{pm}$. (i) Its magnitude E_{pm} is proportional to rotor speed ω_r. So for low wind speeds, if we maintain the optimal tip-speed ratio, we expect this internal voltage to be small. As the wind speed increases, this voltage will rise. Therefore, the power electronics converter that is connected to the stator must be capable of generating a wide range of voltage levels to accommodate this change. (ii) Its magnitude is also dependent on the current that flows in the stator, and in particular it depends on the d-axis component of the current. So, it does not only depend on the strength of the permanent magnets, but on the operating condition as well.

EXAMPLE 5.1. Consider a 750-kW, 16-pole, 600-rpm permanent magnet machine whose parameters are: $L_d = 0.6$ mH, $L_q = 0.8$ mH, $\lambda_{pm} = 1$ V-s, and $R_s = 2$ mΩ. The machine is rotating at rated speed. It is connected to a power electronics converter that acts as a current source and generates $I_{qs} = 300$ A and $I_{ds} = 100$ A. Determine the terminal voltage $\tilde{V}_s$, the internal voltage $\tilde{E}_{pm}$, the stator current $\tilde{I}_s$, and the real and reactive power generated by the machine.

First, we find the rated rotor speed, which is

$$\omega_r = (2\pi)(\text{rpm}/60)(P/2) = (2\pi)(600/60)(16/2) = 502.4 \text{ rad/s}$$

It is easy to calculate the magnitude of the internal voltage:

$$E_{pm} = \tfrac{1}{\sqrt{2}}(502.4)\left[1 - (0.6 - 0.8)(10^{-3})(100)\right] = 362.4 \text{ V}$$

but we do not yet know its angle δ. Now if we multiply both sides of (5.1) by $e^{-j\delta}$ we get

$$\begin{aligned}
\tilde{V}_s e^{-j\delta} &= \tilde{E}_{pm} e^{-j\delta} - (R_s + j\omega_r L_q)\tilde{I}_s e^{-j\delta} \\
&= E_{pm} - (R_s + j\omega_r L_q)\tfrac{1}{\sqrt{2}}(I_{qs} - jI_{ds}) \\
&= 362.4 - (2 + j(502.4)(0.8))(10^{-3})\tfrac{1}{\sqrt{2}}(300 - j100) \\
&= 333.5 - j85.1 = 344.2\angle -14.3^\circ \text{ V}
\end{aligned}$$

Since by definition $\tilde{V}_s = V_s\angle 0$, we obtain $\tilde{V}_s = 344.2\angle 0$ V, and $\delta = 14.3^\circ$. So $\tilde{E}_{pm} = 362.4\angle 14.3^\circ$ V, and

$$\tilde{I}_s = \tfrac{1}{\sqrt{2}}(I_{qs} - jI_{ds})e^{j\delta} = \cdots = 223.6\angle -4.1^\circ \text{ A}$$

The real and reactive power generated can be found by

$$S_s = 3\tilde{V}_s\tilde{I}_s^* = (3)(344.2)(223.6\angle 4.1^\circ) = 230.3 \text{ kW} + j16.6 \text{ kVAr}$$

If the converters are each 97% efficient, the power that reaches the power system is $(230.3)(0.97^2) = 216.7$ kW. The reactive power, however, does not have to flow through. The grid-side converter can generate (or absorb) an amount of reactive power that is independent of Q_s.

5.2. Wind energy conversion with PMSGs

The wind energy conversion process with a PMSG is similar to what happens with a DFIG. A power vs. wind speed characteristic such as the one shown in Fig. 4.3 is obtained. The main difference is that we are no longer limited by a maximum-slip constraint. So, we can vary the rotating speed over a wider range, while at the same time maintaining optimum tip-speed ratio. In a sense, there is only a B-C segment, whereas the A-B and C-D segments are eliminated. This can help us harvest a little more energy from the wind.

5.3. Controlling a PMSG

The main idea behind the control of a PMSG is obvious once the power output of the generator is computed. We will neglect the small resistance R_s to simplify the calculations. The complex power output is $S_s = 3\tilde{V}_s\tilde{I}_s^*$. Utilizing the expressions (5.1), (5.2), and (5.3) we obtain

$$\begin{aligned}
S_s &\approx 3(\tilde{E}_{pm} - j\omega_r L_q \tilde{I}_s)\tilde{I}_s^* \\
&= 3\tilde{E}_{pm}\tilde{I}_s^* - j3\omega_r L_q I_s^2 \\
&= 3\left\{\tfrac{1}{\sqrt{2}}\omega_r\left[\lambda_{pm} - (L_d - L_q)I_{ds}\right]e^{j\delta}\right\}\left\{\tfrac{1}{\sqrt{2}}(I_{qs} + jI_{ds})e^{-j\delta}\right\} - j3\omega_r L_q I_s^2
\end{aligned}$$

$$= \tfrac{3}{2}\omega_r \left[\lambda_{pm} - (L_d - L_q)I_{ds}\right] I_{qs} +$$
$$+ j\tfrac{3}{2}\omega_r \left[\lambda_{pm} - (L_d - L_q)I_{ds}\right] I_{ds} - j\tfrac{3}{2}\omega_r L_q(I_{qs}^2 + I_{ds}^2)$$

Therefore,

$$S_s \approx \underbrace{\tfrac{3}{2}\omega_r \left[\lambda_{pm} - (L_d - L_q)I_{ds}\right] I_{qs}}_{P_s} + j\underbrace{\tfrac{3}{2}\omega_r \left[\lambda_{pm} I_{ds} - L_d I_{ds}^2 - L_q I_{qs}^2\right]}_{Q_s} \tag{5.4}$$

It can be seen that the real power output can be controlled by adjusting the *qd*-axes stator current components. Remember, these are related to the rotor position, so a position encoder is necessary. In some PM machines, $L_d = L_q$ (for example, in machines were magnets are mounted on the rotor surface). In this case, the real power output is proportional to I_{qs}. The other component of current (I_{ds}) is not necessary, so it is usually kept to zero in order to minimize ohmic loss. In other PM machine designs, typically when the magnets are embedded in the rotor, the *saliency* makes $L_d < L_q$ (yes, this is not a typo; unlike what happens in a traditional salient-pole synchronous generator with field winding, in PM machines the *d*-axis inductance is smaller than the *q*-axis inductance because of the presence of the magnets along the *d*-axis that have low magnetic permeability). In this case, there is an additional component of torque, called the *reluctance torque*, which we can utilize to obtain extra power output. The trick is to allow I_{ds} to obtain positive values.

In fact, there is an infinite number of I_{qs}-I_{ds} combinations that produce a desired power output, conditional on the rotor speed. It is convenient to eliminate the rotor speed, and work with the electromagnetic torque instead, which is $T_e = P_s/\omega_{rm}$, where $\omega_{rm} = \frac{2}{P}\omega_r$:

$$T_e = \tfrac{3}{2}\tfrac{P}{2}\left[\lambda_{pm} - (L_d - L_q)I_{ds}\right] I_{qs} \tag{5.5}$$

Solving for the *q*-axis current yields constant-torque hyperbolas of the form $I_{qs} = a/(b + cI_{ds})$. Three such curves are plotted in Fig. 5.4, but there is only one pair of *qd*-axes currents that will provide a given torque value at minimum ohmic loss, which occurs when the current magnitude is minimum. Alternatively, this means that we want to get the maximum possible torque output for a given current magnitude I_s. This control strategy is called *maximum torque per ampere (MTPA)* operation. Mathematically, we can express this problem as follows:

$$\begin{aligned} \text{maximize}_{I_{qs},I_{ds}} \ & \tfrac{3}{2}\tfrac{P}{2}\left[\lambda_{pm} - (L_d - L_q)I_{ds}\right] I_{qs} \\ \text{subject to} \ & I_{qs}^2 + I_{ds}^2 = I_s^2 \end{aligned}$$

This is an optimization problem with an equality constraint, which can be solved by introducing a Lagrangian multiplier λ (this is just a number and should not be confused with λ_{pm} that has a different physical significance). The Lagrangian function is

$$\mathcal{L}(I_{qs}, I_{ds}, \lambda) = \tfrac{3}{2}\tfrac{P}{2}\left[\lambda_{pm} - (L_d - L_q)I_{ds}\right] I_{qs} + \lambda(I_{qs}^2 + I_{ds}^2 - I_s^2)$$

and the solution is found by the set of equations

$$\frac{\partial \mathcal{L}}{\partial I_{qs}} = 0 \Rightarrow \tfrac{3}{2}\tfrac{P}{2}\left[\lambda_{pm} - (L_d - L_q)I_{ds}\right] + 2\lambda I_{qs} = 0$$
$$\frac{\partial \mathcal{L}}{\partial I_{ds}} = 0 \Rightarrow -\tfrac{3}{2}\tfrac{P}{2}(L_d - L_q)I_{qs} + 2\lambda I_{ds} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow I_{qs}^2 + I_{ds}^2 = I_s^2$$

We can solve the second equation for λ, and then substitute in the first one. This yields after some elementary manipulations

$$I_{qs}^2 = I_{ds}^2 - \frac{\lambda_{pm}}{L_d - L_q} I_{ds} \tag{5.6}$$

This equation describes a hyperbolic curve on the I_{qs}-I_{ds} plane, on which all MTPA points lie. This is plotted in Fig. 5.4. For any given stator current magnitude, we can substitute $I_{qs}^2 = I_s^2 - I_{ds}^2$ and then solve a quadratic equation for I_{ds} to obtain the components of stator current that will maximize the torque:

$$2I_{ds}^2 - \frac{\lambda_{pm}}{L_d - L_q} I_{ds} - I_s^2 = 0 \tag{5.7}$$

Once we know the currents (we also compute $I_{qs} = \sqrt{I_s^2 - I_{ds}^2}$), we can find the torque using (5.5). What we have obtained so far is an MTPA mapping or look-up table from I_s to (I_{qs}, I_{ds}) and finally to T_e. However, for controlling the PMSG we are interested in the inverse mapping, i.e., from T_e to (I_{qs}, I_{ds}), so that we can ask the power electronics to generate these currents. This is very easy to obtain by reversing the original look-up table. Of course, the torque cannot be increased indefinitely. The maximum torque is obtained at the intersection of the MTPA curve with a circle defined by $I_{qs}^2 + I_{ds}^2 = 2I_{s,\max}^2$, where $I_{s,\max}$ represents the maximum permissible rms value of current that can flow through the stator windings and/or the power electronics.

5.4. Exercises

(1) Consider a 500-KW, 16-pole, 600-rpm permanent magnet machine whose parameters are: $L_d = 0.35$ mH, $L_q = 0.7$ mH, $\lambda_{pm} = 0.7$ V-s, $R_s = 6$ mΩ. The machine's rated phase current is 700 A rms, and rated voltage is 440 V line-to-line rms. If the machine is generating rated power at rated speed and rated current, determine: (i) $\tilde{I}_s$, (ii) $\tilde{V}_s$, (iii) $\tilde{E}_{pm}$, and (iv) Q_s.

(2) Repeat problem (1) under the assumption that the machine is generating rated power at rated speed and rated voltage.

(3) Similarly to the current equation (5.3), we can define the qd-axes voltages for a PMSG by

$$\sqrt{2}\tilde{V}_s e^{-j\delta} = V_{qs} - jV_{ds}$$

Show that the equivalent circuit equations in phasor form (5.1)-(5.2) are equivalent to

$$V_{qs} = -R_s I_{qs} + \omega_r(\lambda_{pm} - L_d I_{ds})$$
$$V_{ds} = -R_s I_{ds} + \omega_r L_q I_{qs}$$

(4) Prove that the MTPA curve defined by equation (5.6) is a hyperbola.

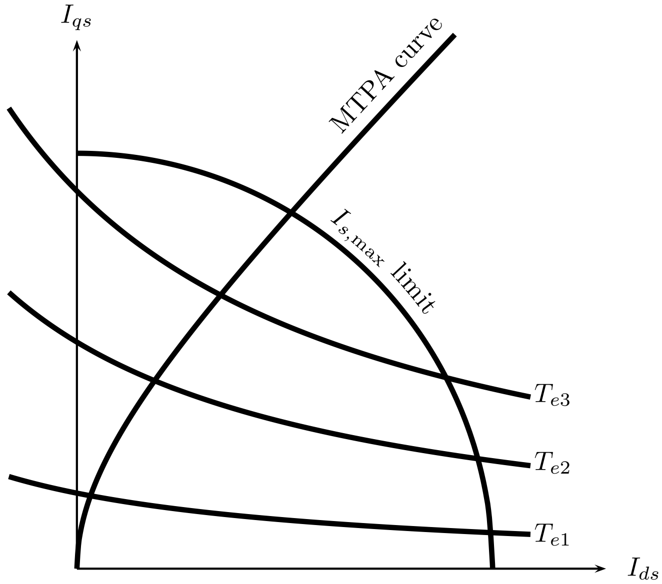

FIGURE 5.4. PMSG characteristic curves on the I_{qs}-I_{ds} plane. Three constant-torque curves with $T_{e1} < T_{e2} < T_{e3}$ are shown. The MTPA curve intersects the constant-torque curves at the points that have minimum distance from the origin.

(5) For the PMSG of problem (1), compute the MTPA mapping $f : \mathbb{R}^+ \to \mathbb{R}^2$, that maps $T_e \mapsto (I_{qs}, I_{ds})$. An analytical expression for f might not exist, so it might be necessary to provide the mapping in the form of a look-up table.

(6) Using the PMSG qd-axes voltage equations of problem (3), show that the condition of constant stator voltage magnitude, $V_{qs}^2 + V_{ds}^2 = (\sqrt{2}V_0)^2$, corresponds to an ellipse on the I_{qs}-I_{ds} plane. (To simplify the analysis, you may set $R_s = 0$.) Sketch a representative ellipse, and explain how its shape changes if ω_r is increased.

(7) For a PMSG, show that the condition of constant reactive power output, $Q_s(I_{qs}, I_{ds}; \omega_r) = Q_0$, corresponds to an ellipse on the I_{qs}-I_{ds} plane. Sketch the ellipse for $Q_0 = 0$, and explain how its shape changes under varying Q_0 and fixed ω_r. Show that the maximum reactive power that a PMSG can output for a given ω_r is

$$Q_{s,\max} = \frac{3\omega_r \lambda_{pm}^2}{8L_d}$$

and that this occurs when $I_{qs} = 0$ and $I_{ds} = \frac{\lambda_{pm}}{2L_d}$. Also explain why the PMSG will absorb reactive power for all (I_{qs}, I_{ds}) combinations, except for the ones that are located inside the $Q_0 = 0$ ellipse.

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