"Design of a Free-Piston Hypersonic Shock Tunnel", by Scott Courtney and Tim J. Alcenius, Fall 1992, AAE590 Project. It would still be nice to build this for teaching purposes, perhaps for 520 or 519, using the student fee funds for equipment costs and the Bruhn fellowship for student labor. The figures and computer code still exist.

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Abstract

The purpose of this paper is to provide a method for designing a low-cost, easy to build, free-piston hypersonic shock tunnel which can be used for educational purposes. This paper will explain the principle of operation and the reasoning behind building such a facility, and present the governing equations for the motion of the piston, shock tube, and expansion of the high temperature gas. The methodology of computing high temperature flow conditions in the shock tube and through the nozzle expansion are discussed, along with validation of numerical coding. The selection of the shock tunnel parameters are explained, and on- and off-design cases are examined. Mechanical design concerns are discussed, along with the actual mechanical design. Design specifications, drawings, dimensions, and all parameters are included for the final design case.

Introduction

Concepts such as heat transfer, non equilibrium flow, ionization, dissociation, and other high temperature effects are very important in the study of hypersonic aerospace vehicles. These effects are very difficult to model theoretically or computationally. There are current facilities existing today which can reach these high temperatures, but they are limited and rare. Free piston hypersonic shock tunnels have been constructed at Caltech and the DLR in Germany, but most of these high temperature ground facilities were very costly to build and expensive to operate and maintain. Consultations with researchers from Australia and Caltech suggest that a small facility could be constructed here at Purdue University for only a few thousand dollars. A low cost facility which could show the various effects of hypersonic ionization and dissociation, along with experimental testing of hypersonic heat transfer effects, would be a very beneficial learning tool for students here at Purdue. A 'hands-on' experience with hypersonics can not only spark interest in the classroom, but possibly become a very beneficial piece of knowledge for students as they enter the aerospace industry.

Overview of Design

The final design will consist of four significant parts:

1) A high pressure compressed air section which will drive a compression piston.
2) A piston compression section which will adiabatically heat and compress a helium driver gas to create a strong shock in the test gas.
3) A shock tube section which will contain the test air gas to be heated and compressed.
4) A test section which will expand the high temperature test air gas through a nozzle to the correct high enthalpy conditions needed to simulate hypersonic flow.

no figures in this word document, only in the paper copy, which I have, SP Schneider, Jan. 2004

Figure 1: General Schematic of Free-Piston Hypersonic Shock Tunnel [1]

Subscripts and the various design parameters are defined as follows:

0	- The initial compression tube conditions
4	- The final compression tube conditions
r	- The compression tube diaphragm rupture conditions
1	- The initial conditions in the shock tube
2	- The conditions in the shock tube after the shock wave
5	- The conditions in the shock tube after the reflected shock wave

a	- Speed of sound (m/s)	U	- Velocity (m/s)
ρ	- Density (kg/m^3)	p	- Pressure (atm)
T	- Temperature (K)	h	- Enthalpy (gcal/g)
A	- Area (m)	L	- Length (m)
d,D	- Diameter (m)	t	- Time (sec)
m	- Mass (kg)	γ	- Ratio of specific heats
M	- Mach number		

Principle of Operation

Hornung [2] suggests that the simulation of hypersonic effects in the laboratory must use a gas flow with the same gas as the prototype to be studied, along with the same flow speed U and the same binary scaling parameter ρL, L being the characteristic length and ρ being the flow density. Since no materials exist that can contain a gas at such high temperatures, the flow will be of short duration. So, the gas must be heated extremely rapidly. Heating and compression of a gas can be done very rapidly by passing a strong shock wave over the gas in a shock tube. By reflecting the shock from the end of the shock tube, the gas can further be heated and compressed. The shock speed can be increased by raising the pressure ratio across the diaphragm of the shock tube, or more powerfully, by raising the speed of sound in the driver gas. This can be seen in Fig. 2.

Hornung states that to achieve Mach numbers of ~20 it is essential to raise the ratio of the speed of sound of the driver gas to the speed of sound of the test gas (a_4/a_1) if excessive pressures are to be avoided. Hornung also states that for a Mach number of 18, for instance, and keeping the pressure ratio below 30,000 (due to the need for high density and difficulty in containing gas at pressures above 2000 atm) that the speed of sound ratio must be at least 10. Thus, a driver gas with a speed of sound of 4 km/s is required if the test gas is initially at room temperature. Noting that helium is a monatomic gas with a low molecular weight and remains a perfect gas at high

Figure 2: Variation of shock Mach number in a constant area shock tube with p_4/p_1 and a_4/a_1 (from [3])

temperatures, and its temperature for a speed of sound of 4 km/s has to be ~ 4600 K. This temperature is too high to be contained for long periods of time, so it must be heated quickly. This can be done efficiently by compressing the gas adiabatically. By using a piston which is pushed into a compression tube containing helium, a temperature of ~4600 K can be reached if the volumetric compression ratio is around 60.[2]

The wave diagram along with a general schematic of a typical free piston shock tunnel can be seen in Fig. 3. The piston is pushed by highly compressed air into a compression tube filled with helium at an initial pressure. The piston then accelerates towards the end of the compression tube, sealed with a high pressure diaphragm. The piston continues to move forward until the helium pressure rises enough to decelerate it, stop it, and cause it to go in the other direction. The diaphragm is chosen such that it bursts shortly before the piston stops so that a constant helium pressure is obtained after the diaphragm has burst. The diaphragm rupture causes the shock wave to be driven into the test gas, and is reflected off of the end of the shock tube thus generating a reservoir of gas at the high enthalpy needed. As the shock arrives at the end of the tube, it bursts a thin diaphragm before the nozzle throat which up to then separates the test gas from the evacuated nozzle and test section. The bursting of this diaphragm allows the high enthalpy reservoir gas to expand through the nozzle, thus converting the reservoir conditions into the desired conditions of the test gas flow [2]

Figure 3: Shock Wave Diagram and schematic diagram for free piston shock tunnel (from [3])

Governing Equations

Piston motion

In his paper, *The Piston Motion in a Free-Piston Driver For Shock Tubes and Tunnels*,[4] Hornung discusses the model equations and derives the parameters important to the piston motion. Typically in the free-piston driver the gas used to accelerate the piston is compressed air. Let the initial pressure in the compression driver air to be p_{oA}, the initial helium pressure be po_{He} .Let the piston speed (positive to the right) be u, the initial distance of the piston from the diaphragm be L, and the variable distance along the compression tube to be x. The helium inertia is neglected, along with dissipation effects caused by vortical flow along the piston-tube wall junction. The piston is assumed to move without friction and seal perfectly against the wall of the compression tube. The reservoir of air to the left hand side of the piston is assumed large enough such that the conditions in the compression driver air are assumed to be subjected to a simple wave, and both the compression driver air and the helium are being considered as a perfect gas.

As derived by Hornung [4]:

The piston driver compression air pressure, p_A, on the left hand side of the piston is given by:

$$p_A = p_{oA}\left(1 - \frac{\gamma_A - 1}{2}\frac{u}{a_o}\right)^{\frac{2\gamma_A}{\gamma_A - 1}} \tag{1}$$

where a_o is the speed of sound in the air and γ_A is the ratio of specific heats in perfect gas air (7/5).

The helium pressure on the right hand side of the piston is given by:

$$p_H = p_{oHe}\left(\frac{x}{L}\right)^{-\gamma} \tag{2}$$

where γ is the ratio of specific heats in helium (5/3).

Using $\Sigma F_i = Ma$, where $F_i = p_i A$, the equation of motion becomes:

$$-M\frac{d^2x}{dt^2} = \frac{\pi D^2}{2}\left[p_{oA}\left(1-\frac{\gamma_A-1}{2}\frac{u}{a_o}\right)^{\frac{2\gamma_A}{\gamma_A-1}} - p_{oHe}\left(\frac{L}{x}\right)^{\gamma} \right] \tag{3}$$

Introducing the dimensionless variables

$$\xi(\tau) = \frac{x}{D} \qquad \tau = \frac{t\,a_o}{D} \qquad \phi(\tau) = \frac{u}{a_o} \tag{4}$$

equation (3) along with initial conditions can be rewritten using a change of variables as

$$\begin{aligned} \dot{\xi} &= -\phi \\ \dot{\phi} &= -b_1\left[(1-0.2\phi)^7 - b_2\xi^{-5/3}\right] \\ \xi(0) &= \frac{L}{D} \\ \phi(0) &= 0 \end{aligned} \tag{5}$$

where the dot denotes differentiation with respect to τ and the parameters b_1 and b_2 are given by

$$\begin{aligned} b_1 &= \frac{\pi}{4}\,\frac{p_{oA}\,D^3}{M\,a_o^2} \\ b_2 &= \frac{p_{oHe}}{p_{oA}}\left(\frac{L}{D}\right)^{\frac{5}{3}} \quad . \end{aligned} \tag{6}$$

Piston Motion Following Diaphragm Burst

Stalker [5] and Hornung [4] both state that it is important to keep the pressure in the main space as nearly as constant as possible. Stalker suggests that conditions in the test section must be maintained constant following shock reflection for the best experimental results. He implies that the conditions in the driver gas (helium) must also be kept constant. Both Stalker and Hornung recommend letting the diaphragm rupture before the compression stroke is complete, or before the piston comes to a complete stop. Stalker states "It is possible to arrange that the velocity of the piston is such that further movement of the piston tends to compensate for loss of the driver gas flowing into the shock tube".[5] Therefore, a constraint equation for the velocity of the piston can be found such that the pressure in the main space is nearly constant.

Using $\Sigma F = Ma$, the equation of motion for the piston after diaphragm burst becomes

$$M \frac{d^2x}{dt^2} = \left(p_{oHe} - p_{Ar}\right)\frac{\pi D^2}{4} \tag{7}$$

The compression ratio at diaphragm rupture is

$$\lambda = \frac{L}{x_r} \tag{8}$$

and the ratio of the speeds of sound of the helium can be related using isentropic relations

$$a_4/a_{oHe} = \left(T_4/T_0\right)^{\frac{1}{2}} = \lambda^{\frac{\gamma-1}{2}} = \lambda^{\frac{1}{3}} \tag{9}$$

If the mass flow rate at sonic conditions is equal to the rate at which the piston displaces mass, then

$$\rho_4\, u_r\, D^2 = \rho^*\, a^*\, d^2, \tag{10}$$

and the rupture velocity over the speed of sound becomes

$$\frac{u_r}{a_4} = \frac{\rho^*\, a^*\, d^2}{\rho_4\, a_4\, D^2} = \left(\frac{2}{\gamma-1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{d^2}{D^2} \tag{11a}$$

with $\gamma = 5/3$

$$\frac{u_r}{a_4} = 0.5625\, \frac{d^2}{D^2} \tag{11b}$$

Therefore,

$$\frac{u_r}{a_{Heo}} = 0.5625\,\lambda^{\frac{1}{3}}\frac{d^2}{D^2} \tag{12}$$

Knowing that the rate of pressure rise at diaphragm rupture is governed by the area ratio d^2/D^2 and the piston speed at rupture, the value of ϕ_r for constant pressure at rupture may be determined from

$$\begin{aligned}\phi_r &= \frac{u_r}{a_o} = \frac{u_r}{a_{oHe}}\,\frac{a_{oHe}}{a_o}\\ &= \frac{u_r}{a_{oHe}}\sqrt{\frac{\gamma_{He}R_{He}}{\gamma_{oA}R_{oA}}} = 2.9333\,\frac{u_r}{a_{oHe}}\end{aligned} \tag{13}$$

Thus

$$\phi_{rc} = 1.65\,\lambda^{\frac{1}{3}}\frac{d^2}{D^2} \tag{14}$$

This relation will constrain the piston velocity at rupture, and in turn will constrain the initial driver parameters.[4]

Another requirement is to have the piston decelerate to rest in the distance that remains after diaphragm rupture x_r, to avoid damage to the piston and the end of the compression tube. It is assumed that the deceleration is uniform in this time, and the pressures on either side of the piston are constant. Noting that the pressure behind the piston is small compared to p_4, the acceleration required to stop the piston over the remaining distance x_r is given by basic kinematic relations with $u_{final} = 0$

$$g = \frac{u_r^2}{2x_r} \tag{15}$$

and from the equation of motion

$$g = \frac{p_4 \pi D^2}{4M} \tag{16}$$

Equating the two, and using u_r from equation (12),

$$(0.5625)^2\, a_{oHe}^2\, \lambda^{\frac{2}{3}} \frac{d^4}{D^4} = \frac{D^2 x_r p_4}{2M} \tag{17a}$$

or

$$P \equiv \frac{p_4\, x_r\, D^2}{4M\, a_{oHe}} = 0.1582 \left(\frac{d}{D}\right)^4 \lambda^{5/3} \tag{17b}$$

gives the parameter P, a constraint which allows for the piston to come to a complete stop after rupture.[3]

Flow Through the Shock Wave

After the high pressure diaphragm ruptures, a normal shock wave will be produced in the shock tube. Due to the high speed of the shock wave, the flow through the shock now has to be considered to be a real gas. This is due to the fact that the shock wave will heat the gas to temperatures at which the molecules will become excited enough to break apart into individual atoms. Therefore, we can no longer use perfect gas relations to complete our system of equations through a normal shock wave.

The three equations for flow through a normal shock are:

$$\rho_1 u_1 = \rho_2 u_2 \tag{18}$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \tag{19}$$

$$h_1 + \tfrac{1}{2} u_1^2 = h_2 + \tfrac{1}{2} u_2^2 \tag{20}$$

We have four unknowns in the above three equations (ρ_2,p_2,h_2,u_2). Therefore, we must be able to find a fourth equation to solve for these variables uniquely. This fourth equation comes from the molecular composition of the gas behind the shock wave.

Equilibrium and Non-Equilibrium Flows

We can get this fourth equation in two ways. The first would be to assume that the reaction rate is much slower than the time that is needed for a particle to travel through the shock wave. This would mean that we would be assuming the flow was a non-equilibrium flow. Then we would have to solve the reaction rate equations for each different species that is present in air as the flow goes through the shock. Since air is a diverse gas, it would take a long time to set up a computer program that would be able to solve these equations. Therefore, this is not a choice that we were willing to make if we did not have to.

The second way would be to assume that the reaction rate is much faster than the time that is needed for the particle to travel through the shock wave. Then we would be assuming that the flow was in equilibrium after going through the shock. If we make this assumption, then a fourth equation is given by Anderson [6].

$$h = \frac{p}{\rho}\left(\frac{\gamma}{\gamma - 1}\right) \tag{21a}$$

where

$$\begin{aligned} \gamma &= c_1 + c_2 Y + c_3 Z + c_4 YZ + \frac{c_5 + c_6 Y + c_7 Z + c_8 YZ}{1 + \exp[c_9(X + c_{10} Y + c_{11})]} \\ X &= \log\left(\frac{\rho}{1.292}\right) \\ Y &= \log\left(\frac{p}{1.013 \times 10^5}\right) \\ Z &= X - Y \end{aligned} \tag{21b}$$

$c_1 \ldots c_{11}$ are given in Anderson[6] p. 463

We can also use these correlations by Anderson to calculate the temperature behind our shock wave. The equation for this is:

$$\log\left(\frac{T}{To}\right) = d_1 + d_2 Y + d_3 Z + d_4 YZ + d_5 Z^2 + \frac{d_6 + d_7 Y + d_8 Z + d_9 YZ + d_{10} Z^2}{1 + \exp[d_{11}(Z + d_{12})]}$$

$$\begin{aligned} X &= \log\left(\frac{p}{1.0134 \times 10^5}\right) \\ Y &= \log\left(\frac{\rho}{1.225}\right) \\ Z &= X - Y \end{aligned} \tag{22}$$

$d_1 \ldots d_{12}$ are given in Anderson[6] p. 464

The accuracy of these equations for high temperature air can be seen from the plot of T versus P for constant ρ (figure 4). The solid lines represent the correlations above, and the tabulated data are calculated using statistical thermodynamics.

Figure 4: Correlation of Equations 21 & 22 to Real Gas Effects (from [6])

Shock Fixed and Laboratory Fixed Coordinates

Now that we have our four equations, we must worry about the correct values for the velocities that we use in our original equations. This is because these equations are assuming that the shock is fixed and the flow has some velocity going into the shock and some velocity coming out of the shock. To account for this, we must fix our shock wave and determine the resulting velocities. This can be done for a normal shock wave as shown by figure 5.

Figure 5: Incident Shock in Shock Fixed Coordinate System [7]

This gives us resulting velocities for the incident shock:

$$V_1 = u_S \qquad (23a)$$
$$V_2 = u_S - u_2$$

Since we are dealing with a shock wave that will be reflected off of the end of our tunnel in order to further increase our stagnation conditions, we must also find the resulting velocities into and out of the shock after reflection. This can also be seen in figure 6.

Figure 6: Reflected Shock in Shock Fixed Coordinate System [7]

This gives us resulting velocities after shock reflection:

$$V_{R2} = u_2 + u_R \tag{23b}$$
$$V_{R5} = u_R$$

Now, we have all of the information that is needed to solve these equations in order to get our stagnation conditions for the nozzle flow.

Methodology of Code

Although we have four equations and four unknowns, our fourth equation is a complicated relation for h, p, and ρ. Therefore, we must use an iteration scheme in order to solve for the unknowns. The scheme used is a very simple iteration method for ρ_2.

The first step is to solve equation (18) for u_2.

$$V_2 = \frac{\rho_1 V_1}{\rho_2} \tag{24}$$

Now we can substitute this into equations 19 & 20 to give the following relations for p_2 and h_2.

$$p_2 = p_1 + \rho_1 V_1^2\left(1 - \frac{\rho_1}{\rho_2}\right) \tag{25}$$

$$h_2 = h_1 + \frac{V_1^2}{2}\left[1 - \left(\frac{\rho_1}{\rho_2}\right)^2\right] \tag{26}$$

For the incident shock wave, all we need to do is to substitute the resulting velocities and known conditions (p_1,h_1) into the above equations, and then solve iteratively for our conditions behind the shock (p_2,ρ_2,u_2,h_2). The methodology of the code written to do this is relatively simple.

1) Guess a value for $\frac{\rho_1}{\rho_2}$ (A good initial guess is 0.1).
2) Solve equations 25 & 26 for p_2, h_2.
3) Calculate a new value for ρ_2 using equations 21 and values of p_2 and h_2 just calculated. Since ρ_2 is needed to calculate h in this equation, use the old value of ρ_2 when calculating X.
4) With this new value for ρ_2, recalculate $\frac{\rho_1}{\rho_2}$.
5) Take this new value and start again with step 2. This scheme should yield convergence in less than 10-15 iterations.
6) Calculate the value for V_2.
7) Calculate the value for T_2 using the given equilibrium equation.

This same scheme can be used to calculate the values for the flow behind the reflected shock also. There are a few modifications that must be made when doing this however. First, in the original equations, all ones must be replaced with twos (since all of the variables behind the reflected shock are now known and are used for inflow into the reflected shock wave), and all twos must be replaced with fives. The second thing that has to be changed is the solutions for p_5 and h_5. Originally, it was sufficient to substitute for V_2 in the above equations because V_2 was unknown and V_1 was known (u_S-u_2 and u_S respectively). However, now u_R is unknown, making both V_{R2} and V_{R5} unknowns. Fortunately, u_R can be solved for in terms of ρ_2, ρ_5, and u_2 and we can replace V_{R2} and V_{R5} in equations 23b. This means that we now can solve our three shock relation equations for u_R, p_5, h_5, and the variable being iterated over ρ_5. These equations are given below:

$$u_R = \frac{\rho_2 u_2}{\rho_5 - \rho_2} \qquad (27)$$

$$p_5 = p_2 + \rho_2 u_2^2 \left(\frac{\rho 5}{\rho 5 - \rho 2} - \rho_2 \right)$$

(28)

$$h_5 = h_2 + \frac{1}{2} u_2^2 \left(\frac{\rho 5 + \rho 2}{\rho 5 - \rho 2} \right) \qquad (29)$$

These equations must be iterated over ρ_5, but the above technique can still be used. The only thing that needs to be done is to calculate ρ_5 in step one and eliminate step four. This yields a simple technique that can be used to find the stagnation conditions for the nozzle flow.

Equilibrium Quasi-One-Dimensional Nozzle Flow

The first thing that must be considered when doing equilibrium nozzle flow is whether or not the chemically reacting flow is isentropic or not. It can be assumed that the flow is both inviscid and adiabatic, and since equilibrium is assumed the first and second laws can be written as (derivation from Anderson [6]):

$$T\,ds = dh - \nu\,dp \qquad (30)$$

We have for an adiabatic steady flow:

$$h_o = h + \frac{1}{2}u^2 \tag{31a}$$

or in differential form:

$$dh + V\,dV = 0 \tag{31b}$$

From the momentum equation for and inviscid, high temperature, equilibrium flow, written along a streamline:

$$dp = -\rho V\,dV \tag{32a}$$

which can be rearranged as:

$$V\,dV = -\frac{dp}{\rho} = -\nu\,dp \tag{32b}$$

All of these equations hold for a high temperature, chemically reacting, inviscid, adiabatic flow since they are all derived from laws of nature. These laws are namely: global conservation of mass, Newton's laws, and thermodynamic laws, which are all independent of the type of gas used, chemically reacting or not.

Now we can substitute V dV into our differential form for dh:

$$dh - \frac{dp}{\rho} = dh - \nu\,dp = 0 \tag{33}$$

Substituting this back into our original equation for ds:

$$T\,ds = dh - \nu\,dp = 0 \tag{30}$$

This shows that chemically reacting, equilibrium nozzle flow is isentropic. This will be an important result when the exit conditions are being calculated for this flow.

We will also be interested to know whether or not sonic conditions occur at the throat for an equilibrium, chemically reacting, quasi-one-dimensional nozzle flow. Since the flow is isentropic, the derivation for the area-velocity relation can be found [8]. This relation is:

$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u} \qquad (34)$$

This relation holds for general gas flowing through the nozzle. It can be seen that when M = 1, dA/A = 0, which means sonic flow does exist at the throat for a chemically reacting, equilibrium nozzle flow.

The next step in our process was to determine the flow conditions at the nozzle throat and the nozzle exit. This was done iteratively using a Mollier diagram. The Mollier diagram is a graph of enthalpy versus entropy with lines of constant temperature, pressure, and density for a given gas (air for our case). Since we have proven that the flow is isentropic, the solution for the throat and exit conditions will lie on a vertical line drawn through the point corresponding to the stagnation conditions. The process for using the Mollier diagram to find the throat and exit conditions is outlined below.

1) Find the point corresponding to h_o, T_o, P_o, ρ_o (known conditions h_5, T_5, P_5, ρ_5 from shock tube calculation).
2) Use the fact that at the throat $\rho V = \rho^* V^* = \rho^* a^* = (\rho V)_{max}$
3) Guess a value of h below h_o (flow conditions are always less than stagnation)
4) Use that h in the following equation to find ρV (derived from $ho = h + \frac{1}{2}V^2$)

$$\rho V = \rho\sqrt{2(h - h_o)} \qquad (31c)$$

5) Guess a new value for h and return to step four. Do this until a maximum value for ρV is found.
6) Read all values at $(\rho V)_{max}$. These will be throat conditions h^*, P^*, T^*, and ρ^*
7) Use conservation of mass from the nozzle throat to the exit. This gives:

$$\rho_e V_e = \rho^* V^* \frac{A^*}{A_e} \qquad (35)$$

8) Choose a value of h below h*. (Since there is sonic velocity at the throat, and we wish to expand to faster than sonic conditions at the exit, h_e will be less than h*).

9) Use the same idea as in steps four and five, this time trying to match ρV from step seven.

10) Read all values at $\rho_e V_e$. These will be exit conditions h_e, P_e, T_e, ρ_e, and V_e.

Going throughout this process will give the conditions through the nozzle. More importantly, it will give the exit velocity and the exit pressure. Knowing these values will tell what the test section velocity will be and what pressure the test section needs to be evacuated to in order not to have shock waves occurring before the test device.

Validity of Code

Piston Motion

Using parameters given in Hornung[4], results of the 4th Order Runge-Kutta Scheme, figure 7, can be compared to results shown in figure 8, given by Hornung.

Figure 7: Solution for b1=0.015, b2 = 20, L/D = 100

Figure 8: Hornung's Solutions [4]

Stagnation Condition Calculation

There are two main ways that could have been used to validate the written code for the calculation of stagnation conditions. The first would be to use a Mollier diagram and find the point corresponding to h, P, T, and ρ. However, after looking at the Mollier diagram, it is evident that gaining a solution from the Mollier diagram is not a greatly accurate process. Since this would have to be done twice (once for flow through the incident shock and once for flow through the reflected shock), it is possible that errors could arise in the code that would not be caught.

The second way would be to compare our results with A. S. Predvoditelev[8] who has compiled thermodynamic properties of air in a large pressure and temperature range. Since this process is a little easier, and much more accurate, this is what we chose to do. Below is a table that gives our computed values compared to Predvoditelev.

Table 1 : Code Results Versus Theory For High Temperature Air [8]

	Calculated	Predvoditelev
Incident Shock		
P_2	49.57 atm	50.0 atm
T_2	8249.60 ^oK	8250 ^oK
h_2	5353.94 $\frac{gcal}{g}$	5330.5 $\frac{gcal}{g}$
Reflected Shock		
P_5	331.55 atm	330.0 atm
T_5	10689.51 ^oK	10700 ^oK
h_5	8206.30 $\frac{gcal}{g}$	8110.4 $\frac{gcal}{g}$

As can be seen from this table, the code calculates accurately the value for the flow variables behind both the incident shock and the reflected shock. This means that the stagnation quantities can be assumed to be known for the initial nozzle flow. The differences between the solutions could be due to the fact that the flow is not entirely in equilibrium through the shock waves or possibly numerical error that arises in the code. In any case, the difference is small enough that we can say that the equilibrium assumption holds for this flow. Hopefully, we will get a chance to actually measure these quantities after the facility is built.

Results

Solution of Piston Motion

The solution of the differential equations in equation (5) can be solved using a 4th Order Runge-Kutta Scheme. Since the design compression ratio was determined from exit conditions to be 60.0, the differential equations can be solved and iterated to find the value of p_{oA} which satisfies the ϕ_{rc} constraint of wanting constant pressure in the main space. the parameter P in equation 17b can be solved to obtain an expression for the piston mass:

$$M = 1.58027 \frac{D^6 x_r p_4 \pi}{a_{He_o}^2 d^4 \lambda^{5/3}} \tag{17c}$$

where x_r is the distance the piston is from the end of the compression tube at rupture. By comparing the piston mass to the mass calculated using equation 17c, the mass required to avoid piston/compression tube end damage can be found. Iterating over p_{oA} gives a compression air pressure of 95 atm, and iterating over M gives a mass of 0.7 kg at design conditions.

All important parameters are now known:

$L = 10$ feet	$D = 2$ inches
$p_{oA} = 95$ atm	$p_{oHe} = 0.1$ atm
$a_{oA} = 340.3$ m/s	$a_{oHe} = 1001.5$ m/s
$M = 0.7$ kg	

The parameters b1 and b2 given by equation 6,

$$b_1 = \frac{\pi}{4} \frac{p_{oA} D^3}{M a_o^2}$$

$$b_2 = \frac{p_{oHe}}{p_{oA}} \left(\frac{L}{D}\right)^{\frac{5}{3}} \tag{6}$$

can be computed using the above parameters, giving

$b_1 = 0.00213$ $b_2 = 11.3258$

Figure 9 shows the Solution to the differential equation for the piston motion for these parameters.

Figure 9: Solution of Piston Motion, b1 = 0.00213, b2= 11.3258, L/D = 60.0

Solution of Gas Dynamics

Figure 10 is a plot of pressure versus compression ratio for our design. As can be seen, P_4 is the most important pressure parameter for safety reasons. We do not want to have extremely high pressure at the end of the compression tube since it would be extremely costly and defeat our original purpose. Therefore, we want to keep the compression tube pressure below 1200 atm, which we feel can be obtained both safely and cost effectively. Since no gas is contained for a long period of time in any area, the temperature was only a minor concern to us when choosing our compression ratio.

This will give us the design compression ratio for our facility. We did not want to choose a compression ratio right at design pressure of 1200 atm because of the error that results from diaphragm burst pressures. Since burst pressure is hard to predict accurately with materials used, we felt that choosing a burst pressure a little lower than 1200 atm would give us enough safety margin in case the diaphragm burst at a higher pressure than that for which it was designed. Knowing this, we chose a compression ratio of 60 which yields a compression pressure ratio of 1090 atm. This compression ratio will give a shock mach number of 19.6 in the shock tube and Lukasiewicz[10] states that this will give us a approximate run time of 90 microseconds.

As can be seen from figures 11-14, this choice of compression ratio will define the rest of the parameters throughout the tunnel. From figure 11, the pressure behind the incident shock (P_2) is about 50 atm and the pressure behind the reflected shock (P_5) is about 350 atm. In figure 12, temperatures at different locations through the entire tunnel is shown. At a compression ratio of 60, temperature at the end of compression (T_4) is about 4500 ^oK, temperature behind the incident shock (T_2) is about 8000 ^oK, and the temperature behind the reflected shock (T_5) is about 10,500 ^oK.

After the stagnation conditions were known for our design case, we could go through and calculate the conditions throughout the nozzle. An important parameter of choice here is the area ratio (A^*/A_e). As can be seen in equation 35, larger values for area ratio will provide larger values of $\rho_e V_e$, and therefore higher exit velocities. However, due to size limitations for both cost effectiveness and tunnel location requirements, the exit area could not be too large. Also, since the throat size must be smaller than the shock tube itself, there was a physical limitation on the throat area . To meet both of these requirements, an area ratio of 100 was chosen. This provided a nozzle throat radius of 0.1 inch and an exit radius of 1 inch, which were determined to be reasonable to meet our physical limitations.

The next step was to use the Mollier diagram to find the conditions throughout the nozzle. Since using the Mollier diagram is both difficult and time consuming for determining nozzle conditions, we only chose four compression ratios at which to calculate our values. These compression ratios were 40, 50 , 60 , and 70. As can be seen in figures 13 & 14, the data is not completely accurate. This is due to the fact that the Mollier diagram is very difficult to read accurately. However, we are only looking for general numbers and trends for conditions through the nozzle since it will never be possible to determine these conditions with any great accuracy. See appendix A for full computational solution.

Selection of Shock Tunnel Parameters

Given certain physical, mechanical, and cost concerns of the compression tube, the initial sizing of the components are fairly fixed. Since the cost of this facility is of major concern, physical sizes of compression tubing should be minimized. By using standard stock sizes for pipe, cost can further be reduced. Standard pipe sizes should be considered for honed tubing (required for the compression tube), with adequate wall thicknesses for high pressure usage. One such standard pipe size has an inside diameter of 1.993/2.000 with a wall thickness of 0.188 inches. Smaller tubing required for the shock tube does not require a honed inside diameter, but a standard size for special smooth I.D. is .560/.565 inches with a wall thickness of .155 inches. Therefore, the values for d and D are now set.

The tube lengths should also be minimized for cost reasons. The length of the compression tube is constrained only to be long enough for the piston to adequately accelerate to desired burst conditions. The shock tube should theoretically be as long as possible, since the run time of the shock tunnel is dependent solely on this, but the cost concern outweighs the need for extended run times. The compression tube length , L, and shock tube length were both selected to be 10 feet.

Due to the fact that high pressure gas sources require large and expensive storage devices, the compression air pressure should also be minimized. Since high pressure vessels are expensive and require added mechanical pumps, regulators, etc. the decision to use standard tanks of air was made. The unregulated air pressure in standard tanks run about 3000 psi. (~200 atm.) which gives a maximum for p_{oA}. Since the gasses will be initially at standard temperature, a_{oHe} and a_{oA} are set at 1001.5 m/s and 340 m/s respectively. The piston rupture speed constraint, ϕ_{rc} , allows the compression air pressure to be calculated, since all other initial driver conditions have been set.

Non-Optimal Design Case

Since the design diaphragm burst pressure can not be obtained consistently due to the unreliable nature of high pressure diaphragms, an off-design case must be studied. Any change in diaphragm burst pressure may cause the pressure in the main space to fluctuate. Also, the piston speed may become high enough that damage occurs to the piston and/or the end of the compression tube. The diaphragm burst pressure can be determined to about 30% accuracy, so by running several different cases, the effect of changes in diaphragm burst pressure can be studied. Worst-case occurs when compression air pressure is +30% and diaphragm rupture pressure is -30%. By using P_{oA} = 123.5 atm and P_4 = 763.0 atm, the speed of the piston as it hits the end of the compression tube is about 163 m/s.

Using collision analysis, the force of the piston hitting the end of the tube can be found.

$$F = M_1(v_{n1} - v_{n1}') \tag{36}$$

$$vn1' = \frac{M_1 - e\,M_2}{M_1 + M_2} v_{n1} + \frac{(1 + e)M_2}{M_1 + M_2} v_{n2} \tag{37}$$

where

v_{n1} = initial speed of particle 1 (piston)
M_1 = mass of particle 1 (piston)
v_{n2} = initial speed of particle 2 (mass of compression tube)
M_2 = mass of particle 2 (end of compression tube)
e = Coefficient of Restitution (assumed to be 0.8)

With v_{n1} = 163 m/s, M_1 = 0.7, v_{n2} = 0 m/s M_2 assumed to be 500kg ,

$$F \sim 205 \text{ Newtons} \sim 45\ lb_f$$

which is within limits to avoid damage.

Mechanical Design

There are 4 main concerns with the mechanical design of the hypersonic shock tunnel.:

1) Safety in the containment of the high pressures that are created.
2) Parts that are easily available and easy to machine to reduce cost.
3) Components that can be worn out must be easy to replace.
4) A final design which is easy to operate and maintain.

With the high pressures that are created in this facility, it is the number one priority to consider safety. Parts that are purchased from supply companies must satisfy all maximum pressure requirements, including worst case off-design. Parts which are to be machined must also meet these requirements, with appropriate factors of safety to compensate for design tolerances. Another important aspect to consider is cost. Components that have to be specially made or ordered or parts that require a lot of shop time to machine add to the cost of the facility. Using standard size parts can reduce cost and are readily available from most companies. Components that are subject to increased wear must be easy to replace. Extra replacement parts are needed for both maintenance reasons and also for the initial "shakedown" of the facility, where small design changes may be needed. These extra parts must also be considered in the cost of the components. Finally, the facility must be easy to operate and maintain. The operation of the facility should be considered, allowing for easy installation of test objects, easy replacement of diaphragms, and an efficient method of running the facility. Expected maintenance should be considered and kept to a minimum, as this will add cost to the facility as time goes on.

From the gas dynamic design, many component parameters have been set. Since the compression tube and shock tube diameters were selected to correspond to standard pipe sizes, the fittings and other components are also easily available in stock parts. The following figure shows an overview of the final mechanical design (see figure 16).

Standard pipes and pipe flanges were selected from a Kilsby-Roberts catalog, and O-ring specifications were selected using a National O-Rings catalog. All cylinder tubing is class DOM, ASTM spec. A-513, type 5, yield approx. 70,000 psi. All class 600 pipe flanges are rated at 1,500 psi working pressure and all 150 class pipe flanges are rated at 290 psi working pressure. 8 inch diameter carbon steel stock material for high pressure section is rated at 30,000 psi yield.

Compression Air Driver Tube

The high pressure compression air is to be held in 2 1/2 inch O.D., 1.997/2.000 inch I.D. honed steel cylinder tubing, 0.252 inch wall. The compression air driver tube will be 10 feet long to avoid any wave interactions that may occur when the piston expansion wave travels down the compression air tube and reflects back. Two inch standard class 600 pipe flanges will be used to seal the far end of the compression air tube, one blind flange and one slip-on flange welded to one end of the cylinder tubing. (See figure 17). An o-ring groove will be cut into the slip-on flange before welding, with an outside gland diameter of 3.257" (+ .005, -.000) with a gland width of 0.21" (+.010, -.000) and a gland depth of 0.107" (+ .005,-.000). [O-Ring Dash # -232]. The two flanges will be bolted together using eight 5/8 x 4.25 inch stud bolts. The design compression air pressure is 95 atm, 1,396 psi, well below the quoted 70,000 psi yield for the tubing and below the rated 1,500 psi for both flanges.

Trigger Diaphragm Section

The other end of the high compression air tubing will have the same 2 inch class 600 slip-on flange welded to the end with the same o-ring grooves cut into it. Two 1/16 inch diameter holes will drilled on top and bottom, offset at 1.88 inches from the center of the flange for diaphragm dowel pin holders (see figure 18). The front end of the flange will also have 1/8 inch diameter dowel pin holes bored in them to a 0.25 inch depth. The 1/16 inch hole will be used to tap out any dowel pins that may break off inside the flange. A second 2 1/2 inch O.D. honed steel cylinder tubing, 0.252 inch wall, 9 feet 4 inches, will be used for the piston compression section and will contain the helium and piston. The end of this tubing will have the same 2 1/2 inch class 600 slip-on flange welded to it with the same 3.257 x 0.21 x 0.107 o-ring groove and the same 1/16 & 1/8 inch holes drilled and bored in the front face. The two flanges will be bolted together using eight 5/8 x 4.25 inch stud bolts. An aluminum diaphragm will be used and scored to burst at the design pressure of 95 atm. The burst pressure (1396 psi) is below the rated pressures for both the tubing and the flanges.

High Pressure Compression Section

Due to the high pressures that result from the piston compression, two 8 inch carbon steel pieces will be used to contain the high pressure. From the plot of compression tube pressure vs. distance from the compression tube end, figure 19, it can be seen that the pressure rises drastically as the piston reaches the end of the compression tube. Due to the fact that the class 600 pipe flanges are rated to only 1,500 psi, and for safety reasons, the last portion of the compression must take place inside a stronger vessel. Figure 19 shows at 8 inches from the end of the compression tube, the pressure reaches 1,500 psi. Therefore, an 8 inch piece of carbon steel stock with 2.00 inch diameter hole bored into it will be used to contain the high pressure on the compression tube side. With the design high pressure diaphragm burst pressure at 1096 atm, 16,019 psi, the hoop stress will require that the wall thickness be at least 1/2 inch for the carbon steel at 30,000 psi yield. Making the total diameter 8 inches gives a wall thickness of 3 inches and a factor of safety of 6, which will be adequate to account for the drilled holes needed to attach the flange and adequate to account for the needed safety of the facility. The 2 1/2 inch class 600 flange will be welded on to the end of the compression tube with the same 3.257" x 0.21" x 0.107" o-ring groove. The carbon steel stock will also have eight 5/8 inch bolt holes at a bolt circle diameter of 5.00 inches to thread the stud bolts through for connection (see figure 20) . The other end of the compression tube carbon steel stock piece will have the same 3.257" x 0.21" x 0.107" 0-ring groove, with two 1/4 inch dowel pin holes bored in to a 0.375 inch depth, offset from the center 1.88 inches. The shock tube carbon steel stock piece will have a 0.56 inch diameter hole in it, and eight 5/8 inch diameter holes drilled through it. It also has the same 3.257" x 0.21" x 0.107" o-ring groove and 1/4 inch dowel pin holes on the compression tube side. Eight 0.62 inch holes bored and tapped out to a 1 inch depth will be used to attach the shock tube to the carbon steel stock. Both pieces of carbon steel stock will be faced smooth on the ends that hold the diaphragm for a tighter seal. An aluminum diaphragm will be used and scored to burst at the design pressure of 1090 atm.

Shock Tube Section

The shock tube will be made of 7/8 inch O.D. steel cylinder tubing, unhoned, special smooth, 0.56 inch (+.000, -.010) I.D., 0.155 inch thickness, 9 feet 8 inches long. Both ends will have a 1/2 inch class 600 slip-on pipe flange welded to the ends with

1.444 inch outside diameter, 0.21 inch width 0.107 inch depth o-ring groove cut into it. [O-Ring Dash #-216] The test section side of the shock tube will have two 1/16 and 1/4 inch holes drilled and bored to a 0.25 inch depth into the flanges for diaphragm dowl pins as before (see figure 21). A mylar diaphragm will be used to separate the shock tube section from the test section. The incident shock pressure is 49.6 atm, 728.5 psi, well below the tube yield, and the reflected shock pressure is 331.5 atm, 4871.7 psi, which is above the 600 class flange rating, but the mylar diaphragm provides a release for the pressure.

Test Section

The test section will consist of a 6.625 O.D. standard weight straight tee with five 6 inch class 150 slip-on pipe flanges welded on each side with 7.757 inch O.D., 0.21 inch width. 0.107 inch depth o-ring grooves cut in them. [O-Ring Dash #-264] (see figure 22). The top and one end of the straight tee will be closed with 6 inch class 150 blind pipe flanges. The far end blind flange will have a 0.56 inch hole tapped and drilled to mount the test object sting tubing. The 'top' blind flange will be drilled to facilitate a glass or Plexiglas window for viewing. A 6 1/2 inch cylinder tubing 8.5 inches will have two 6 inch class 150 slip-on flanges with the 7.757" x 0.21" x 0.107" O-ring groove cut into them. This tubing will house the length of the nozzle such that the exit of the nozzle and test object will be centered with the top opening of the straight tee. A 6 inch class 150 blind pipe flange will close the test section and it will have a 0.56 inch diameter hole drilled in the center with two o-ring grooves, one 7.757" x 0.21" x 0.107" and the other 5.132" x 0.021" x 0.107" [O-Ring Dash#-249] with two 1/16 and 1/4 inch holes drilled and bored to a 0.25 inch depth offset at 0.875 inches from the center of the flanges for diaphragm dowel pins(see figure 21). Eight 0.62 diameter holes will be drilled in the blind flange to connect the shock tube end to the test section. the bolts will go through the 1/2 inch flange and the 6 inch flange to attach to the nozzle and seal the test section.

Conical Nozzle

The conical nozzle is made of 6 inch aluminum stock, 12 1/2 inches long. A 1.5 inch diameter hole is bored 4 inches deep to house a steel nozzle throat insert (see figure 21). Eight 0.62 inch diameter holes are drilled and tapped into the aluminum stock 1 inch deep. The conical portion of the nozzle will start out 4 inches down from the end at

0.56 inch diameter and the end will be at a 2 inch diameter exit to keep the conical angle at 5 degrees. The nozzle throat piece will be of slightly smaller diameter than 1.5 inches and a 0.20 inch hole will be drilled through to make the nozzle throat diameter. (Hornung suggests that the throat diameter be about 1/3 of the shock tube diameter). The throat will then be rounded to 0.56 inch inside diameter at the ends and 0.20 inch diameter at the throat. This piece will set inside the aluminum nozzle and then the nozzle assembly will be bolted to through both the inch class 150 flange and the shock tube end 2 inch class 600 flange. The test section will be sealed by bolting the 6 inch flange to the test section (see figure 23).

Experimental Set-Up

The main purpose of having the facility that has been designed is to be able to do something productive with it. Since this will be a laboratory facility, experiments need to designed that will be useful in showing principles of hypersonic flow.

One issue in keeping the cost of our facility down was to use a conical nozzle instead of a contoured nozzle. This meant that the flow in our test section would not be parallel to the test object and therefore would be of no use in showing flow around the entire body. However, the streamline that goes through the center of the nozzle throat will continue to be straight up to the test object. This means that this line in the flow (and some others relatively close to it) will be useful in doing experiments.

One area of great concern to vehicles such as the space shuttle is the heat transfer to the nose as it reenters the atmosphere. Using the straight streamline in our nozzle flow, it would be possible to experimentally determine the heat transfer to the body and then compare it to the theoretical value. This could be done by the use of a heat transfer gauge on the nose of the test object. The idea behind this gauge being that it will measure the heat transfer to the nose with some delay. Then the slope of the heat transfer line could be found and this would determine the heat transfer rate. Using this principle, the heat transfer rate could easily be found without having to worry about buying very costly high speed electronics to keep up with a flow that is on the order of one hundred microseconds long.

Theoretically, calculating the heat transfer rate is a difficult problem, and therefore poses a obstacle to doing this type of experiment. The full equation for the heat transfer for a sphere is given by [6]:

$$q_w = .763\, Pr^{-0.6} (\rho_e \mu_e)^{0.5} \sqrt{\frac{du_e}{dx}} (h_{aw} - h_w) \tag{38}$$

where

$$Pr = \frac{\mu c_p}{k} \tag{39}$$

$$\frac{du_e}{dx} = \frac{1}{R} \sqrt{\frac{2(P_e - P_{inf})}{\rho_e}} \tag{40}$$

However, viewing these three equations together, it is obvious that

$$q_w \sim \frac{1}{\sqrt{R}} \tag{41}$$

This means that the stagnation point heating is inversely proportional to the square root of the radius. This is a very important concept in the design of hypersonic vehicles and is the reason why these vehicles have blunt noses and not sharp leading edges. Therefore, this would be a very good concept for students to see experimentally and also be easy to design.

The experiment could be done as follows. First, two pressure transducers would be needed to measure the speed of the shock wave in the shock tube so stagnation conditions could be found. This would be to insure that the flows in the different runs would be approximately the same. Two or three blunt bodies with different nose radius would then be tested and the heat transfer would be measured. These results would then be plotted against nose radius and the slope of this line would be measured. Assuming that the flows in each of the different runs were approximately the same, the above relation should be found.

This would give students a very unique insight as to why hypersonic vehicles use blunt noses and not sharp ones. We also believe that the students would gain something out of this experiment that they would not otherwise by just doing the theory. In future projects, it would be possible to have some students design a numerical code that would calculate the above heat transfer exactly. This could also then be used as part of the teaching tool in this whole process.

Conclusions

The goal of designing a hypersonic wind tunnel with a relatively inexpensive price for student use has been achieved. This design for a free-piston type tunnel will provide high velocity flows around test objects for actual flow measurement. Although the flow quality in such a tunnel would not be very good, the actual ability for students to "get their feet wet" in real hypersonic flow would be an immeasurable benefit to an educational experience. We have shown that a very beneficial experiment could be set up in the facility, and students would be able to learn here something that otherwise would not be available to them. Although this design only considered student use, it will be noted that in the future, upgrades could be made to the facility to provide the better flow quality and longer flow times that would be needed for hypersonic research.

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